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A New Method for Predicting the Solar Heat Gain of Complex Fenestration Systems
I. Overview and Derivation of the Matrix Layer Calculation

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A New Method for Predicting the Solar Heat Gain of Complex Fenestration Systems
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Abstract

A new method of predicting the solar heat gain through complex fenestration systems involving nonspecular layers such as shades or blinds has been examined in a project jointly sponsored by ASHRAE and DOE. In this method, a scanning radiometer is used to measure the bidirectional radiative transmittance and reflectance of each layer of a fenestration system. The properties of systems containing these layers are then built up computationally from the measured layer properties using a transmission/multiple-reflection calculation. The calculation produces the total directional-hemispherical transmittance of the fenestration system and the layer-by-layer absorptances. These properties are in turn combined with layer-specific measurements of the inward-flowing fractions of absorbed solar energy to produce the overall solar heat gain coefficient.

In this first in a series of related papers describing the project, the assumptions and limitations of the calculation method are described and the derivation of the matrix calculation technique from the initial integral equations is presented.

Introduction

Solar heat gain has become an increasingly important and complex aspect of the continuing effort to make windows more energy-efficient. On one hand, the usual strategy of reducing summer cooling loads by limiting solar heat gain has been complicated by the increasing recognition that the use of daylight with proper lighting controls can produce substantial energy savings in commercial buildings, a strategy that argues for high visible transmittance. Building energy simulation studies (Choi, Johnson et al. 1984) show that window management yields both peak and annual energy savings, and in fact assume that some form of shading for glare control is necessary for any successful daylight utilization. On the other hand, recent measurements (Klems 1989; Klems 1992) show that solar heat gain is an important determinant of the winter energy performance of windows. While sophisticated (i.e., building simulation model-based) calculations of window performance would include the effect of winter solar gain, all too frequently discussions of winter performance have been based on simplified calculations that considered only changes in U-value. The effect of solar gain on winter performance is particularly important in residences, where a large percentage of windows have some form of shading or privacy device. Thus, it is not possible to calculate window performance either in winter or summer without considering solar gain, and it is necessary to address the issue of solar gain through windows with nonspecular shading devices (such as shades, blinds, etc.), which we term complex fenestration systems.

The traditional method of determining complex fenestration system performance is by measurement in a solar calorimeter, (Parmelee, Aubele et al. 1948; Parmelee, Aubele et al. 1953; Ozisik and Schutrum 1959; Pennington, Smith et al. 1964; Yellott 1965) but a systematic characterization by this method poses some daunting problems. Shades, blinds and drapes vary widely in reflectance, transmittance, and color. Moreover, the optical properties of venetian blinds vary with slat tilt angle. All of them may be combined with glazings of various numbers of panes, pane thicknesses, tints and coatings. To construct a solar heat gain rating system analogous to the NFRC U-value method, (NFRC 1991) it would be necessary for a manufacturer to determine the performance of a product line (possibly containing many products that differ only in color and surface pattern) in combination with every possible glazing system (as well as every adjustment configuration for systems such as venetian blinds). To do this by calorimeter measurement for each distinct combination would require a prohibitive amount of testing. On the other hand, to construct an analytical model of the type that has sometimes appeared in the literature (Farber, Smith et al. 1963) for each specific type of shading device would be a large research effort, for which much of the essential heat transfer data are not currently available.

Therefore, it is useful to devise a method of calculating solar heat gain that is intermediate between the extremes of calorimetric measurement and first-principles calculation, enabling one to calculate the solar heat gain through complex fenestration systems from a smaller and more easily obtained set of measurements. We have been developing and validating such a method in a research project sponsored jointly by ASHRAE and the U.S. Department of Energy. This paper begins the summary of that work with the derivation of the calculation method by physical arguments. A companion paper (Klems 1993B) presents a more detailed account of the use of the method in calculating solar-optical properties for multilayer systems.

A New Method for Calculating Solar Heat Gain

We begin by examining the usual expression for the solar heat gain coefficient, F, of a fenestration system,

$$F = \tau + N_I \alpha , \tag{1}$$

where τ and α are the transmittance and absorptance, respectively, and N_I is the inward-flowing fraction of the absorbed solar energy. If we recognize that this quantity inherently depends on the solar incident angle, θ, that for a device which is not cylindrically symmetric (such as a blind, which has a preferred direction, the slat orientation) it may also depend on the angle, ϕ, between a characteristic direction and the plane of incidence, and that the fenestration consists of M layers denoted by i, then we can generalize Equation (1) to make it valid for any fenestration system:

$$F(\theta,\phi) = T_{fH}(\theta,\phi) + \sum_{i=1}^{M} N_i A_{fi}(\theta,\phi), \tag{2}$$

where T_{fH} denotes the *front directional-hemispherical transmittance of the system* , and N_i, A_{fi} denote the inward-flowing fraction and front absorptance, respectively, of the i^{th} *layer*. The *layer inward-flowing-fraction*, N_i, represents the fraction of the energy absorbed in the i^{th} layer that ultimately flows into the building space, and is the analog of N_I in equation 1. We next observe that T_{fH} and A_{fi} are purely optical quantities, depending on such properties as wavelength and reflectance, but not on temperature. The layer inward-flowing fraction N_i is the only truly calorimetric quantity in the equation. It may depend on the geometry of the fenestration system, as well as on temperatures and other heat transfer variables, but does not depend on the short-wavelength properties of the system, such as color or reflectance.

It follows that if N_i can be determined separately a single determination will be useful for specifying the solar heat gain coefficient for a large number of systems. For example, the values of N_i for a double glazed system with an interior shade should apply to all colors and patterns of shades and to clear, tinted and even some coated glazings. Coatings that modify the emissivity, however, would require a different set of N_i values. We therefore took the approach of measuring N_i for a series of thermally prototypic system geometries and combining these measurements with detailed optical measurements of the components to produce the solar heat gain coefficient of an individual system.

We can summarize the essentials of the method as follows: We begin with the optical properties of individual layers, which are derived from measurement. For specular layers such as glasses these properties are frequently known or may be obtained using standard optical techniques. For non-specular layers biconical optical measurements are required, and it was necessary to develop an apparatus and a technique for measuring them, as described below. A mathematical technique was developed to combine the individual layer properties to produce the overall system transmittances and layer absorptances appearing in equation 2. This method turns out to be most conveniently implemented utilizing matrix equations analogous to a previously-published treatment (Papamichael and Winkelmann 1986). A conceptual outline of the method has also been presented previously, (Papamichael, Klems et al. 1988) and the complete derivation appears below.* The system optical properties are then combined with measurements of N_i for thermally prototypical geometries. The determination of F as a function of incident direction for a particular device in a given fenestration system thus requires

- Measurement of the bidirectional (or biconical) transmittances and reflectances of the nonspecular device,
- Knowledge of the solar-optical properties of the other layers of the system (e.g., glass properties),
- Calorimetric measurement of the layer inward-flowing fractions N_i for the particular geometric and thermal system configuration under consideration,
- Calculation of the system directional-hemispherical transmittances and layer-by-layer absorptances, and
- Calculation of the solar heat gain coefficient, $F(\theta,\phi)$, using equation (2).

Calculation of System Optical Properties

Our approach to the optical problem begins with a consideration of transmission of incident radiation through a generalized optical system to a point P, as indicated in Figure 1(A). Because the system is not assumed to be specular, point P may receive radiation from any points in the system, indicated by the rays from Y and Y' in the diagram. In general, the radiation ariving from two points, Y and Y', will not be the same. This may be for either of two reasons. The optical properties of the system may differ at the points Y and Y', or the transmission may depend on the angle through which the incident ray is scattered. Since point P is at a finite distance from the optical system, the scattering angles θ_1 and θ_1' will differ. In the general case, the scattering direction must of course be specified by two angles (as must the incident direction), but for the

* The reader should note that while the general matrix form presented in the references (Papamichael and Winkelmann 1986) and (Papamichael, Klems et al. 1988), is preserved in the present work, the detailed mathematical expressions in the references do not distinguish between outgoing radiance and incoming irradiance. This work makes that distinction, resulting in different mathematical expressions.

present we will use a single angle to denote direction in order to simplify the notation of the discussion. The second angle will be included at the appropriate time.

The effects of angle dependence and sample inhomogeneity are separated in Figure 1(B). If we consider two points P and P' connected to the points Y and Y' by parallel rays, differences in the radiation arriving at the two points will be solely due to spatial inhomogeneities. If we consider that the optical system consists of more than one layer (neglecting for the moment the possibility of interreflections between layers), for any path of a ray Y_1Y_2P through the system to point P, we can construct a translated image ray $Y_1'Y_2'P'$ that passes through a given point Y' in the initial layer, as shown in Figure 1(C). If the incident radiation flux is denoted $I(\theta_0)$, where θ_0 denotes the direction of the incident radiation, then the radiation arriving at point P will be given by

$$I(P) = \int\int d\boldsymbol{\theta}_2 d\boldsymbol{\theta}_1 T_2(Y_2, \theta_2; \theta_1) T_1(Y_1, \theta_1; \theta_0) I(\theta_0), \tag{3}$$

where the boldface symbols indicate that the integral is a two-dimensional one. We note that for a given point P, the points Y_1 and Y_2 are related to the directions θ_1 and θ_2 by the requirement that radiation originating at Y_1 must arrive at point P; therefore only two of the four quantities are independent, and we have chosen the directions in equation 3. The functions T_1 and T_2 are the transmittances of the two layers. Equation 3 expresses the fact that one must add up radiation arriving at P by all possible paths (still neglecting interreflections). The radiation arriving at the translated image point P' would be given by exactly the same expression, except that Y_1 and Y_2 would be replaced by the translated points Y_1' and Y_2'. The total transmission of the system would then be obtained by adding up I(P) for all points lying on a suitable surface subtending the interior side of the optical system (for example, a hemisphere or an infinite plane). In principle, the incident radiation should be treated spectrally, expressing the intensity at a particular wavelength as $I(\theta_0,\lambda)$, and the calculation of equation 3 should be done at each wavelength and the result integrated over wavelength.

Thus, for an exact treatment one would need to characterize each layer of the optical system by a transmission function $T_i(Y_i,\theta_i;\theta_{i-1},\lambda)$, where θ_{i-1} is the direction of the radiation emerging from the previous layer (or the direction of the incident radiation, for i-1=0). Since, as has been noted, both the point Y_i and the directions θ_i and θ_{i-1} are inherently two-dimensional quantities, each layer would need to be specified by a transmission that is a function of seven variables. This is an impracticably large amount of information and leads one to ask to what extent approximations and simplifications are possible to reduce the information requirements.

The first approximation is to use optical properties that are spectrally averaged over the transparent region of glass, 300-2700 nm, neglecting the region of low transparency between 2700 nm and the point, around 4500 nm, where it becomes effectively opaque. As is well known, (Rubin 1985) the spectral transmittance of clear glass is relatively flat in the 300-2700 nm region, with the highest (and most constant) transmittance in the visible. As we show in Appendix 1, the condition for a treatment utilizing integrated spectral properties to be valid is that most layers have spectrally flat optical properties, with at most one strongly selective layer. The spectrally-averaged method should be suitable, then, for fenestrations systems consisting of clear glass layers and non-selective shading elements, with at most one tinted or selective layer. As the method is applied to progressively more complicated systems, such as colored blinds or shades combined with tinted or selective glazing, or multiple blind or curtain systems of different colors, the treatment could become less accurate and eventually inapplicable. As will become clear, this research has been focused on the spatial and angular aspects of systems rather than the spectral properties. Our studies have been confined to the spectrally simple end of this continuum, and further work will be necessary to determine the limits of applicability.

The second approximation is to remove the explicit dependence on Y in equation 3 by replacing the transmission functions in equation 3 by averages taken over a suitably-sized area, with the incoming and outgoing directions held fixed. This is the equivalent of averaging together the energy arriving at point P in Figure 1(B) with all of its translated image points as indicated by point P' in that figure. This will not change the total transmittance, which depends only on the sum of I(P) over all such points. The information lost in this averaging process is that which would enable one to construct an image of the fenestration system as seen from a particular point P or P'. This is a level of detail not important to the problem at hand.

The approximation in effect provides a way of treating systems with important spatial inhomogeneities, such as venetian blinds, as though they were homogeneous layers. The key point is that the essential inhomogeneity is of a limited size, for example, a slat size of one to two inches in a venetian blind. The entire device consists of repetitions of this basic unit, so if one has averaged properties over a spatial area large compared to this unit, one obtains a property that applies (on the average) anywhere on the device. This is the analog of treating diffuse surfaces (which are inhomogeneous on a very small spatial scale) as uniform materials. In this approximation, the directly-transmitted radiation from a venetian blind in the sunlight would be a uniformly-lit patch. In reality, a venetian blind produces a series of stripes, and in addition a series of small dots corresponding to the holes where the blind cords penetrate the slats. The difference between the two treatments does not become important until one is making calculations that depend on knowing radiant fluxes with spatial resolution on the order of fractions of an inch. This is far beyond the present or forseeable future requirements of building energy calculations.

Applying this spatial averaging to each layer does place limitations on the applicability of the method to certain systems. In Figure 1(C) it can be seen that if the averaging process described is carried out for the entire system, then the overall total transmission is unaffected, but if it is carried out separately for each individual layer additional information is lost, namely the correlation between the points Y_1 and Y_2 (or Y_1' and Y_2' for the translated image ray). The calculation will not be correct for systems for which this correlation is important. This means that the method will not be applicable to systems containing two layers with spatial inhomogeneities of comparable size in the same dimension. For example, a system with two identical venetian blinds would not be correctly treated: With the blinds in a partially closed position, one could change the overall transmittance from a very high to a very low value by translating one of the blinds in its plane by half a slat width in the direction perpendicular to the slats. On the other hand, when the inhomogeneities are not of comparable size, such as a system containing both a venetian blind and a drape, or when the inhomogeneities are in different dimensions, such as a horizontal and a vertical venetian blind, positional correlations between layers should not be important and the method will be applicable. Inapplicability to a small and rather unlikely class of fenestration systems seems a small price to pay for the great advantage of simplicity provided by this approximation.

With these two approximations, we are left with a description of the fenestration in terms of the bidirectional optical properties of successive layers. We now proceed to make the language of equation 3 more precise. We define the wavelength-averaged solar-optical properties of the i^{th} layer in a fenestration in terms of its bidirectional transmittance and reflectance distribution functions (Nicodemus 1965) as follows:

$$I(\theta_i,\phi_i) = \tau_i^f(\theta_i,\phi_i;\theta_{i-1},\phi_{i-1})E(\theta_{i-1},\phi_{i-1}), \tag{4a}$$

where (θ_i, ϕ_i) represents the outgoing direction of the radiation, $(\theta_{i-1}, \phi_{i-1})$ the incoming direction, and $E(\theta_{i-1}, \phi_{i-1})$ is the irradiance (energy per unit area) incident on the front surface of the layer by radiation going in the incident direction in the +z hemisphere. The quantity τ_i^f is the front bidirectional transmittance distribution function of the layer, and $I(\theta_i, \phi_i)$ is the radiance (energy per unit area per unit solid angle) of the radiation emerging out of the back side of the layer in the outgoing direction, which is in the +z hemisphere. In the coordinate system shown in Figure 2, the z axis is the outward normal to the back side of the layer. We will make our diagrams with the front sides of layers on the left, so that we will sometimes refer to radiation into the +z hemisphere as right-moving. Layers are numbered from front to rear. Reflectance from the front side of the layer produces an outgoing radiance, denoted by J,

$$J(\theta_i^r, \phi_i^r) = \rho_i^f(\theta_i^r, \phi_i^r; \theta_{i-1}, \phi_{i-1}) E(\theta_{i-1}, \phi_{i-1}) \tag{4b}$$

consisting of radiation in the reflected direction (θ_i^r, ϕ_i^r), which is in the -z ("left-moving" or "backward") hemisphere, where ρ_i^f is the front refectance distribution function for layer i. Since the layer cannot be assumed to be front-back symmetric, there are analogous relations for radiation incident on the back side:

$$J(\theta_i^r, \phi_i^r) = \tau_i^b(\theta_i^r, \phi_i^r; \theta_{i+1}^r, \phi_{i+1}^r) E^r(\theta_{i+1}^r, \phi_{i+1}^r), \tag{4c}$$

$$I(\theta_i, \phi_i) = \rho_i^b(\theta_i, \phi_i; \theta_{i+1}^r, \phi_{i+1}^r) E^r(\theta_{i+1}^r, \phi_{i+1}^r), \tag{4d}$$

where E^r denotes the back-side irradiance from left-moving radiation in the direction $(\theta_{i+1}^r, \phi_{i+1}^r)$ (the subscript denoting that this radiation comes from the i+1st layer), and τ_i^b, ρ_i^b are the back transmittance and reflectance distribution functions, respectively, of the ith layer. The incident irradiance may be calculated from the radiance emerging from the adjacent layers as follows:

$$dE(\theta_{i-1}, \phi_{i-1}) = I(\theta_{i-1}, \phi_{i-1}) \cos(\theta_{i-1}) d\Omega_{i-1}, \tag{5a}$$

$$dE^r(\theta_{i+1}^r, \phi_{i+1}^r) = J(\theta_{i+1}^r, \phi_{i+1}^r) \cos(\theta_{i+1}^r) d\Omega_{i+1}^r, \tag{5b}$$

where $d\Omega_{i-1} = \sin(\theta_{i-1}) d\theta_{i-1} d\phi_{i-1}$ and similarly for $d\Omega_{i+1}^r$. Thus, the equation analogous to equation 3 for calculating the irradiance emerging from a pair of layers (still without considering interreflectances between layers) becomes

$$I(\theta_2, \phi_2) = \int d\Omega_1 \cos(\theta_1) \tau_2^f(\theta_2, \phi_2; \theta_1, \phi_1) \tau_1^f(\theta_1, \phi_1; \theta_0, \phi_0) E(\theta_0, \phi_0), \tag{6}$$

and the energy per unit area arriving at point P is simply

$$E(P) = \int I(\theta_2, \phi_2) \cos(\theta_2) d\Omega_2, \tag{7}$$

reproducing more precisely the two integrations that were indicated schematically in equation 3.

It is possible to translate this language of functions and integral equations into a more tractable language involving matrices. The details of this translation are given in Appendix 2. Essentially, the forward and backward hemispheres are divided into a finite grid of directions, shown in Figure 3, and the radiances and irradiances are averaged over the finite elements of solid angle associated with each direction and placed in a column vector using the ordering indicated in the figure. This converts angular information into position in a vector or matrix. In this treatment the radiance function I and J become column vectors **I** and **J**, which refer to entire distributions of radiance over outgoing angle. An equation, such as 5a, which refers to a single outgoing direction becomes a statement about a single element in a vector. The irradiance functions E and E^r likewise become column vectors **E** and $\mathbf{E}^r$, and the (front) transmittance and reflectance distribution functions τ_i^f and ρ_i^f become matrices, $\boldsymbol{\tau}_i^f$ and $\boldsymbol{\rho}_i^f$, each element of which represents a biconical transmittance (or reflectance) for a particular incident direction (row number) and outgoing direction (column number). The integrals, such as equation 4a, in the above treatment become matrix multiplications such as

$$\mathbf{I}_i = \boldsymbol{\tau}_i^f \cdot \mathbf{E}_{i-1}, \tag{8}$$

which may be interpreted as a transmittance operator (matrix) for layer i operating on an incident irradiance vector (from layer i-1) and converting it into an outgoing radiance vector emerging from (the back side of) layer i. In the course of this treatment new quantities, called propagation operators, (which are diagonal matrices) appear.* These transform radiance vectors emerging from one layer into irradance vectors incident on the next layer, e.g.,

$$\mathbf{E}_{i-1} = \boldsymbol{\Lambda} \cdot \mathbf{I}_{i-1}. \tag{9}$$

These matrices are in reality geometrical quantities associated with the partitioning of solid angle, and hence do not depend on the layer or whether the radiation is forward-moving or backward-moving.

In this new language equation 6 becomes the matrix equation

$$\mathbf{I}_2 = \boldsymbol{\tau}_2^f \cdot \boldsymbol{\Lambda}_1 \cdot \boldsymbol{\tau}_1^f \cdot \mathbf{E}_0, \tag{10}$$

which can be read as the incident irradiance's undergoing transmission through layer 1, propagating to layer 2, being transmitted through layer 2 and emerging as an outgoing radiance. (We are still neglecting interreflectances between layers.) Note that vectors and matrices are ordered *from right to left* in the order in which the radiation encounters the fenestration layers. One can think of this equation as defining a two-layer system transmittance matrix,

$$\text{``}\ \mathbf{T}_{2,\{1,2\}}^f = \boldsymbol{\tau}_2^f \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{\tau}_1^f\ \text{``}, \tag{11}$$

for which

* It is in the appearance of these propagation matrices that this treatment differs from that of (Papamichael and Winkelmann 1986). The reader should compare equations (9), (10), and (13) with equations (5), (7), (8) and (10) of that reference.

$$\mathbf{I}_2 = \mathbf{T}^f_{2,\{1,2\}} \cdot \mathbf{E}_0 . \tag{12}$$

The quotation marks surrounding equation 11 are intended as a reminder that it is not a correct expression for the system transmittance because it does not include multiple reflections. In (Klems 1993B) it is shown that the correct equation for the two-layer front transmittance is

$$\mathbf{T}^f_{2\{1,2\}} = \tau^f_2 \cdot \left(\mathbf{1} - \mathbf{\Lambda} \cdot \mathbf{\rho}^b_1 \cdot \mathbf{\Lambda} \cdot \mathbf{\rho}^f_2\right)^{-1} \cdot \mathbf{\Lambda} \cdot \tau^f_1 , \tag{13}$$

where the new quantity with a -1 superscript appearing is the inverse matrix of the expression in parentheses, and accounts for the infinite series of non-specular multiple reflections between layers 1 and 2.

We have thus shown in this example how, with a few simplifying assumptions, it is possible to develop a mathematical method for deriving the overall system transmittance matrix of a set of layers, including all the effects of multple interreflections between layers, from the optical properties of the individual layers. One can similarly calculate the system biconical reflectance matrix. The directional-hemispherical transmittance as a function of angle, which is the quantity needed for equation 2, appears (for the selected directional grid) as the elements of a row vector given by

$$\mathbf{T}^{f,H}_{2\{1,2\}} = \mathbf{u}^{\mathrm{T}} \cdot \mathbf{\Lambda} \cdot \mathbf{T}^f_{2\{1,2\}} , \tag{14}$$

where $\mathbf{u}^{\mathrm{T}}$ is a row vector that has each element equal to 1. Multiplying any matrix on the left by the quantity $\mathbf{u}^{\mathrm{T}} \cdot \mathbf{\Lambda}$ is equivalent to an integration over the outgoing hemisphere. The system front directional absorptance, which is the other optical quantity necessary for equation 2, similarly appears as the elements of a row vector that can be calculated using the same method. These calculations are carried out in detail in (Klems 1993B).

One can see from equation 13 that optical properties matrices are necessary for radiation incident on both the front and back side of the layers, and that layers are not assumed to be symmetrical. This means that the derivation of 13 depends only on the "input-output" properties of the layers and not on their internal nature. Either set of layer matrices could inself be a subsystem property matrix describing a set of several layers (for example, N). Equation 13, then, provides the basis for computing the transmittance of a system of N+1 layers by adding an additional layer to a known system of N layers. It should not be surprising that it is possible to develop recursion relations that allow one to build up system properties for an arbitrary number of layers. This is also done in (Klems 1993B).

It is therefore possible, given the biconical properties of the component layers, to calculate the quantities $T_{fH}(\theta,\phi)$ and $A_{fi}(\theta,\phi)$ for any fenestration system.

Conclusion

This paper has begun the description of an ASHRAE/DOE research project to define a method of determining the solar heat gain of complex fenestration systems. It has been shown that with suitable approximations the properties of individual fenestration layers can be characterized by spatially averaged bidirectional (or biconical) reflectances and transmittances, and that the very complicated integral equations governing the propagation of solar radiation through a multiple layer system can be transformed into equations relating matrices made up from the layer properties. It has been asserted that this technique allows the correct inclusion of multiple reflections, and expression for the transmittance of a two-layer system has been given, and it has been asserted that one can build up the properties of a system composed of an arbitrary number of specular or non-specular layers using the same techniques. These assertions are substantiated in a companion paper that discusses the mathematics of the technique more fully. Subsequent papers describing the measurement of layer properties, the determination of inward-flowing fractions, and comparison of the solar heat gain factors determined by the method with calorimeter measurements are planned.

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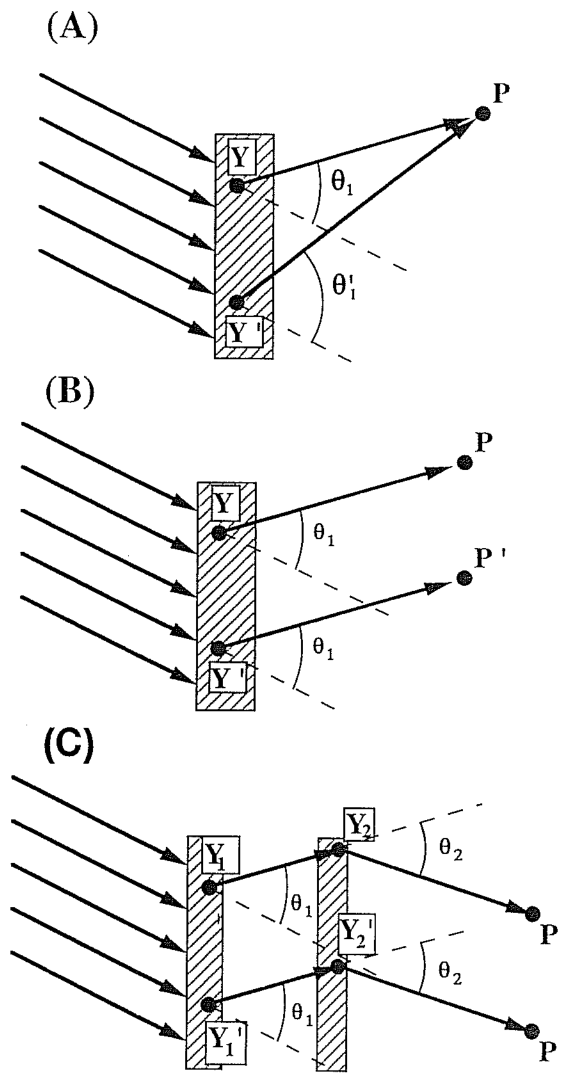

Fig. 1. Transmission of Radiation Through a Generalized Optical System. (A) Radiation may arive at an arbitrary point P from various points Y, Y'. (B) Fixing the scattering direction at each point defines the translated image point P'. (C) For a system consisting of multiple layers, the path to the translated image point P' is an optical path parallel to the original one.

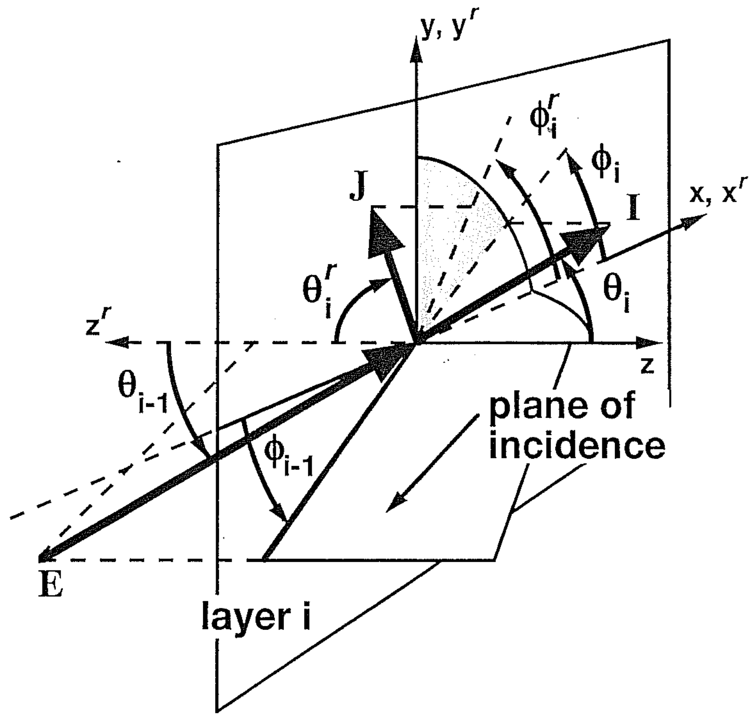

Fig. 2. Definition of the Coordinate Systems for a Layer. The incident irradiance $E(\theta_{i-1},\phi_{i-1})$ and the forward-going (i.e., transmitted) radiance $I(\theta_i,\phi_i)$ are described in the xyz coordinate system, while the backward-going (i.e., reflected) radiance $J(\theta_i^r,\phi_i^r)$ is described in the reflected coordinate system $x^r y^r z^r$, which is left-handed. All quantities with a superscript r refer to the latter. For the general case I and J may have any direction in their respective hemispheres, as indicated. For specular radiation both I and J would lie in the plane of incidence, with $\theta_i^r = \theta_i$. Note that the forward and backward coordinate systems are related by a reflection through the xy plane, so that in that plane they represent the same two spatial axes viewed from opposite sides.

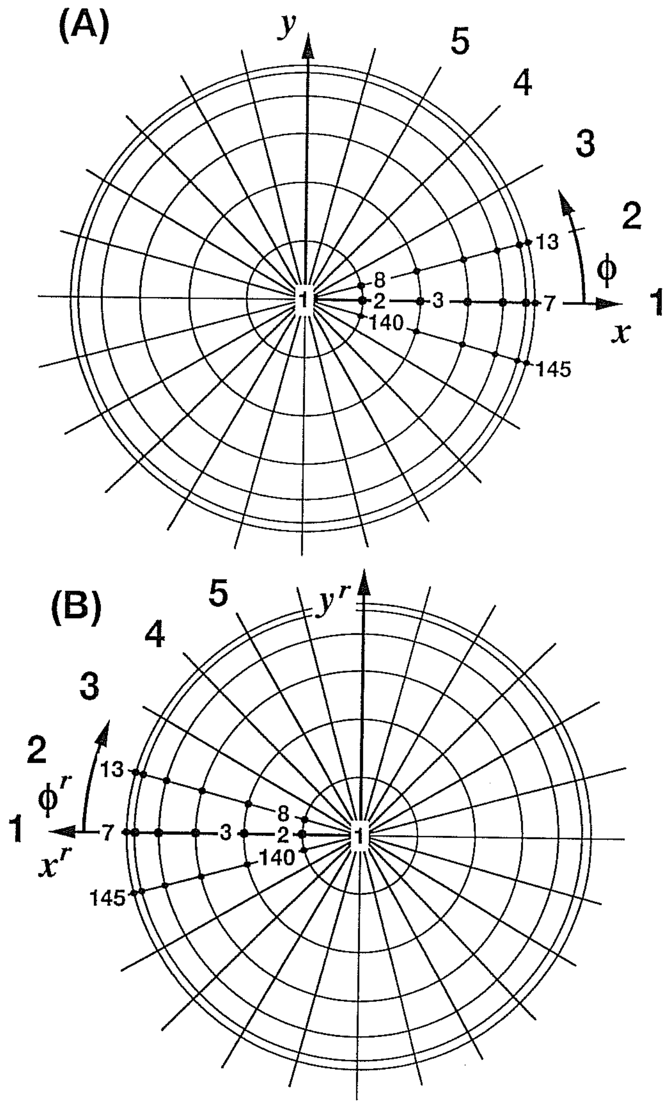

Fig. 3. Angle Coordinates for Incident, Transmitted and Reflected Rays. (A) Coordinates for incident and forward-going radiation. The angles are refered to the xyz coordinate system of Fig. 2; in this figure the z axis is perpendicular the plane and points toward the viewer. The numbers indicate the ordering of directions in constructing vectors and matrices. (B) Coordinates for backward-going radiation. The angles are refered to the $x^ry^rz^r$ coordinate system in Fig. 2; the z^r axis points out of the plane of the figure.

Appendix 1. Effect of Spectral Averaging

The energy transmitted by a pair of layers (neglecting layer interreflectances for simplicity) for radiation with a spectral density I(λ) (energy per unit wavelength) will be given by

$$I_T = \int d\lambda \cdot T_1(\lambda) \cdot T_2(\lambda) \cdot I(\lambda). \tag{A1.1}$$

If we denote the wavelength-integrated incident radiation by

$$I_0 = \int d\lambda \cdot I(\lambda) \tag{A1.2}$$

and define the spectrally-integrated mean transmittance of each layer by

$$\langle T_1 \rangle = \frac{1}{I_0} \int d\lambda \cdot T_1(\lambda) \cdot I(\lambda) \tag{A1.3}$$

for layer 1 and a corresponding expression for layer 2, and if we define the spectrally-integrated mean transmission of the pair of layers by

$$\langle T_1 \cdot T_2 \rangle = \frac{1}{I_0} \int d\lambda \cdot T_1(\lambda) \cdot T_2(\lambda) \cdot I(\lambda), \tag{A1.4}$$

then to replace the wavelength-by-wavelength treatment of transmission by a wavelength-averaged one, it must be true that

$$\langle T_1 \cdot T_2 \rangle = \langle T_1 \rangle \cdot \langle T_2 \rangle. \tag{A1.5}$$

We must therefore inquire under what conditions equation A1.5 fails.

It is easy to see that if one of the transmissions is a constant, equation A1.5 will always hold. If, for example, $T_1(\lambda)=T_{10}$, a constant, then the constant factors out of equation A1.3, leaving $\langle T_1 \rangle = T_{10}$, and also out of the integral in equation A1.4, leaving $\langle T_1 \cdot T_2 \rangle = T_{10} \cdot \langle T_2 \rangle$. On the other hand, it is also easy to construct a case where the relationship fails. For example, let λ_0 be the median wavelength of the incident spectrum, and suppose that T_1 is equal to one for $\lambda<\lambda_0$ and zero otherwise, while T_2 is equal to zero for $\lambda<\lambda_0$, and otherwise is equal to one. Then we would have $\langle T_1 \cdot T_2 \rangle = 0$, while $\langle T_1 \rangle = \langle T_2 \rangle = \frac{1}{2}$. From this admittedly artificial case, we see that equation A1.5 fails when there is more than one selective layer in the system if the selective layers have their peak transmission in very different spectral regions.

More generally, if one of the layers is selective and the other layer has an average transmission in the transmission region of the selective layer that is different from its average transmission over the full spectral region, then equation A1.5 will fail and a spectral characterization of each layer (or, alterternatively, a non-spectral measurement of the combination) will be necessary.