

DESIGN CALCULATIONS FOR PASSIVE SOLAR BUILDINGS
BY A PROGRAMMABLE HAND CALCULATOR

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ABSTRACT

The behavior of room temperature in a passive solar building without backup heat is of great interest to the building designer. This paper presents programs for card-reading programmable hand-calculators which compute room temperature over the course of a design day. Instructions for calculating the input parameters, and for running the programs are given, and a brief review of the theory is provided. The program can presently be used only for single-zone unmanaged, direct-gain buildings.

INTRODUCTION

Floating, or non-thermostat-controlled, room temperature in a passive solar building is an important measure of the building's performance. Optimally designed buildings will provide temperature which floats within the comfort range of the occupants without the use of heaters or air conditioners. Even if some supplementary heating or cooling is used, the floating performance of the building is of interest to the designer in assuring that full use is made of the solar energy collected by a building, and that the optimum window area is chosen.

Solar energy collected by a passive solar building is useful only to the extent that it either balances heating loads during sunny periods or can be stored for use at night. If solar heat gain increases room temperature beyond the comfort range, then the excess heat is either lost through ventilation or else it results in discomfort. In either case, a design whose maximum temperature is lower than the designer's upper limit, (e.g., 80°F (26.7°C)) is preferable to one which heats up beyond this limit.

The lowest floating temperature is also of interest to the designer. The magnitude of the minimum, and the time of day at which it occurs, will determine, for a given occupant's thermal preference characteristics, whether supplementary heat is needed.

Tradeoffs can be made in building design which affect different aspects of floating temperature behavior. Increasing the window area increases the maximum room temperature, but may also decrease the minimum temperature. Adding to the building's thermal mass decreases the magnitude of daily fluctuations in temperature without changing the daily average. Thermal mass can also delay the times of temperature extrema; for a Trombe wall, these delays can exceed 12 hours.

This paper presents a hand-calculator program which can be used to predict the floating temperature of a building, given a few simple building parameters and weather data. The calculations describe the building's response to a design day -- that is, a day with idealized (sinusoidal) weather. Two versions of the program are given; one for a Hewlett-Packard HP-67 calculator and the other for a Texas Instruments TI-59. Listings of the program are given in Appendix A.

The programs described here can be run in less than one-half hour; in some cases (e.g., those in which a few parameters are varied from an initial design), the run-time is considerably less. The methodology used in the programs can be generalized beyond the level of detail available in the programs. Some of these extensions can be done as hand calculations using intermediate outputs of the programs.

Use of this program will allow the building designer to easily predict the floating performance of a proposed single-zone, unmanaged passive solar building. The effect on floating temperature of varying parameters such as properties of the thermal mass and area of windows can be seen and, thus, optimal values can be chosen for such parameters.

The theoretical basis of the programs is described in Appendix B, and detailed derivations can be found in Ref. 1. Some familiarity with the theory will be helpful to the user of the programs, as it will assist in evaluating the input parameters to the program. As can be seen, the theory parallels that used in many public-domain building energy use analysis computer programs (e.g., NBSLD², DOE-2^{3,4}, TWOZONE⁵, BLAST⁶), except that Fourier transformations are used instead of Laplace transformations, and some additional approximations are made.

The program has been validated by comparing its predictions with the measurements of floating temperature performed in two passive solar test cells at Los Alamos Scientific Laboratories.⁷ Predictions of the temperature elevation (above average ambient temperature) were accurate to within $\pm 10\%$ of measurements at all hours of the day for two tests days (see figure, page 27).⁸

THEORY: RESPONSE FUNCTIONS

This section describes the use of response functions in the programs. Their derivation is discussed Appendix B. The response functions describe the response of temperatures in the building to driving forces such as sunlight and outside air temperature. They are functions of frequency (denoted by ω); that is, they give the effect of regular variations in the driving forces at a frequency ω on the variation of building temperatures at the same frequency. The most interesting frequency is generally one cycle per day.

The surface temperatures on the inside surface of building elements (e.g., walls and floors) are described by the materials response functions R_1 and R_2 . These are complex-valued functions which are evaluated by the sub-programs for each building element. R_1 describes the response to sunlight inside the room, and R_2 gives the response to outside temperature.

The overall building performance is described by three building response functions A, B, and C, which are computed using the R_1 and R_2 results for each building element. A and C have the form of design heat loss rates. The A function relates the floating of room temperature to heating or cooling loads, while the C function describes the building response to ambient temperature variations. Response to sunlight is given by the B function.

Room temperature as a function of time is calculated using the building response functions A, B, and C, and weather data for a design day.

PROGRAM INPUT PARAMETERS

The passive solar programs require some simple weather inputs, and some data on building parameters. The building parameters are in a form which differs somewhat from other building models, since a simple format which accounts for most of the passive solar effects is desired.

Before any building parameters can be evaluated, the building must be divided into a small number of different types of construction sections. Each construction section is associated with a surface which faces the inside of the building. A typical house, for example, might have envelope wall sections, ceiling, floor, partition walls, and windows.

If a section is of thermally light construction (e.g., a window or a wall or floor with an insulating material on the inside) the 'U-value' (overall heat transfer coefficient) should be calculated, and no further data entries are needed. If a section is heavy (e.g., masonry construction; solid wood, gypsum board), then more detail is needed.

A heavy wall is divided conceptually into two layers: a thermally massive, inside-facing layer (e.g., concrete walls or floor) and an outside insulating layer (e.g., foam insulation plus sheathing or dry soil). For the inside thermally massive layer, data are needed on conductivity, heat capacity per unit volume, and thickness of the massive layer. For the insulating layer, only U-value of the insulation is needed. If there is no exterior insulation, the outside film coefficient is used.

For thermally massive elements connected to the room by an insulating layer, such as face brick on the outside of an insulated wall, the thermal mass of the outer layer is ignored. For walls with two layers of similar but not exactly equal thermal properties, such as drywall or plaster on solid wood, averaged parameters are used as an approximation. For example, a 4-inch wood wall covered on the outside by sheathing and on the inside by 1/2-inch plaster may be considered as a 4-1/2-inch wall with an insulating layer outside whose resistance is equal to that of the sheathing plus the outside air film. The properties of the thermal mass are the weighted average properties of the wood and plaster.*

Walls which cannot be approximated by this two-layer method are beyond the scope of the present programs because different formulas are needed to evaluate their response functions R_1 and R_2 . Hand calculation methods which can compute these response functions are discussed in Ref. 1, appendix 2.4. An example of such a section might be a 1-inch-thick wood panel covering the inside of an 8-inch concrete wall or floor.

The program is fastest to run when the number of different surfaces or sections is minimized. The TI-59 program can only treat three different sections. To reduce the number of sections, different walls or floors with similar thermal properties can be combined into a single section; and sections which are much lighter than the rest of the house can be considered light.

*To make the whole wall's U-value come out correct, one should average the inverse of conductivity rather than conductivity itself.

One can, for example, combine 4-inch frame envelope walls, 8-inch frame ceilings and floor, and 4-inch frame partition walls into a single section (call it a 6-inch frame wall) if the outside layer's U-value (U_r) is adjusted such that the heat transfer coefficient is the same as it would have been if each section were treated separately. (See Appendix E for an illustration of this process.)

In detailed building energy use analysis programs, such as DOE-2, NBSLD, BLAST and TWOZONE, frame walls are considered as two different sections; one with the properties of the solid wood (stud) fraction, and the other with the properties of the insulated section. This approach will also work in the hand-calculator programs, at the expense of adding an extra surface.

Once the number of surfaces has been established, the user must estimate how much of the solar radiation entering the house is absorbed on each surface. The fraction of solar gain absorbed on a surface is designated by α . The sum of α 's for all heavy surfaces is less than one, because some fraction of the sunlight is absorbed by light surfaces or furniture. This fraction is released immediately to the room, and is called α_R .

There are at present no simple methods for determining the values of the α 's from theory, even for simple room geometries. For complex geometries, even the detailed computerized methods break down. Empirical evaluation is a possibility. Fortunately, building performance does not appear to depend crucially on the exact evaluation of these parameters.

As a rough guideline, for a dark floor and light walls and ceiling,
 $\alpha_{\text{floor}} \cong .45$, $\alpha_{\text{envelope walls}} \cong .10$, $\alpha_{\text{partition walls}} \cong .20$, $\alpha_{\text{ceiling}} \cong .10$,
 $\alpha_R \cong .15$. These estimates are based on computer runs using the "Lumen

II" program⁹ to calculate radiation balance for a prototype passive solar room in winter.

If the walls are dark, this changes to: $\alpha_{\text{floor}} \cong .30$, $\alpha_{\text{envelope walls}} \cong .20$, $\alpha_{\text{partition walls}} \cong .35$, $\alpha_{\text{ceiling}} \cong .05$, $\alpha_R \cong .10$.

Following is a list of input parameters needed.

For each massive surface:

- K - The thermal conductivity of the inside massive layer (in Btu/°F-ft-hr). (Note that many handbooks express K in other units, such as Btu-in/°F-ft²-hr).
- ρc - The heat capacity per unit volume (Btu/°F-ft³). This is usually obtained by finding the density ρ and specific heat c from handbook tables, and multiplying these values. $\rho c \approx 9$ for wood and varies from about 15 to 30 for concrete.
- d - The thickness (in feet) of the massive part of the section. For the partition walls use half the wall thickness.
- h - The inside film heat transfer coefficient coupling the surface to the room air (in Btu/°F-hr-ft²). A typical value is 1.5, although sparsely furnished buildings with few partition walls may have lower values (~ 1).
- U_r - The U-value of the resistance between the outside of the massive part of the section and the ambient air (in Btu/°F-ft²-hr). Typically, $U_r \cong 5$ for a bare thermal mass or about 0.1 for an insulated mass. Concrete slab floors on grade with perimeter insulation have $U_r \sim 0.01$. For partition walls, $U_r = 0$.

- α - The fraction of solar energy entering the house which is absorbed on the surface (including multiple reflections). α is dimensionless.
- A - The total area of the surface facing the room (ft^2).

For the whole building:

- \hat{U}_q - The design heat loss of the building per degree temperature difference through all quick heat transfer mechanisms, including window heat loss, infiltration loss, and conduction through quick construction sections (in Btu/°F-hr).
- α_R - The fraction of solar energy entering the house which is absorbed on light surfaces, furniture, carpet, etc. (α_R is dimensionless.)
- H - The daily average heater output plus internal loads (Btu/hr). Typically, internal loads are about 2000 Btu/hr for a residential unit in the United States.

Weather parameters:

- ω_0 - Daily frequency: 2π radians/24 hr.
- t_d - The length of the day (in hours) from sunrise (or time of first solar gain through the window) to sunset (or time of last solar gain).
- $|S_1|$ - Amplitude of daily solar gain through the windows (Btu/hr).
In practice this is obtained by requiring that daily total solar gain is correct; that is, $\int |S_1| \sin \omega_1 t$ equals the daily solar gain.

Thus

$$S_1 = \frac{\pi}{2t_d} \times \text{daily solar gain} .$$

Daily solar gain can be obtained for a sunny day from ASHRAE solar heat gain factors,¹⁰ or it can be approximated by multiplying window

transmissivity by measured solar heat flux on a surface oriented in the same direction as the window. Typical winter transmissivities for south-facing windows are ~ 0.85 for single-pane, 0.75 for double-pane, and 0.65 for triple-pane glass. To model cloudy days, the value of S_1 must be reduced. Solar gain through all windows is considered in computing S_1 . Errors can result if east or west window area is large compared to south window area.

- \bar{T}_A - Average ambient temperature in $^{\circ}\text{F}$.
- $|\Delta T_A|$ - The amplitude of diurnal temperature fluctuations ($^{\circ}\text{F}$) or one-half the difference between maximum and minimum temperature.
- t_{ϕ} - The number of hours between sunrise and maximum ambient temperature. Note that minimum temperature is modeled as occurring 12 hours from maximum, so choose t_{ϕ} for best overall fit of sinusoidal temperature,

$$T_A = \bar{T}_A + \Delta T_A \cos(\omega_0(t-t_{\phi}))$$
 to real temperature.
- \bar{S} - For a weather cycle in which $|S_1|$ varies sinusoidally from day to day, the average value of $|S_1|$, (Btu/hr).
- ΔS_W - For a weather cycle, the amplitude of variation of $|S_1|$ over the cycle, (Btu/hr).
- ω_W - Frequency of weather variations (in radians/hr). ω_W is smaller than ω_0 ; typically $\omega_W \cong .1\omega_0$.
- ΔT_{A_W} - Amplitude of weather-cycle variation in ambient temperature ($^{\circ}\text{F}$).
- t_a - The time in the weather cycle at which the ambient temperature is maximized (hrs).

- t_s - The time in the weather cycle at which solar gain is maximized (hrs).

Results of weather cycle variations are not presently computed in the HP-67 program; however, the response functions can be obtained from the program and the weather cycle response may be computed by hand, as discussed in the HP program description.

PROGRAM OPERATION

This section describes the operation of the program from the point of view of the user. It assumes that the user has already evaluated all the building and weather parameters. A more detailed description of the HP-67 and TI-59 programs is given in Appendix C. A listing of the programs will be found in Appendix A. The HP-67 program and the TI-59 program are different in structure, so they are described separately below. The instructions must be followed exactly to assure correct output.

To check the performance of the programs and the selection of input parameters, a sample problem is set up and solved in Appendix E.

HP-67 Program

This program consists of three sub-programs. The first, sub-program 'R₁₂', calculates the R₁ and R₂ functions for a surface, given the building parameters for its construction section. The functions are evaluated at five frequencies (0, ω_w , ω_o , $2\omega_o$, and $3\omega_o$) and the results are read out on a data card. This program is run once for each surface.

The results on the data card produced by this program are valid for any material surface with the same construction as the one computed, so that

if many runs are to be made on buildings of similar construction, the user may wish to build up a library of R_{12} data cards for commonly used construction sections.

For construction sections beyond the scope of the R_{12} sub-program, response functions can be computed manually and written onto data cards to be used in the rest of the calculation.

The second sub-program, 'ABC', computes the Building Response Functions from the data stored on R_{12} data cards. Each data card is read into the calculator once, some additional data are entered, and the calculator computes the effects of the new surface on A, B, and C. Any number of surfaces may be used. When the ABC sub-program is completed, the results are written on a data card; the card contains A and B (evaluated at all five frequencies) and $C(\omega_w \text{ and } \omega_o)$. Note that $C(0) \equiv A(0)$.

The final sub-program, ' T_R ', takes the data from the 'ABC' sub-program and computes coefficients of $e^{i\omega t}$ from Eq. (B11) in Appendix B. The program next evaluates the temperature for each hour of the design day using (B11). It displays t , the time (relative to sunrise), for one second; then displays $T_R(t)$ for 5 seconds (or prints it); then displays t for the next hour and the new T_R . The first time and temperature displayed correspond to (solar) midnight, the second to 1a.m., etc. The coefficients of Eq. (B11) used to evaluate T_R are retained in memory.

At present, the effects of internal load and heater output must be added manually by reading $A(0)$ from the ABC data card, decoding the entry using subroutine 'd' of the ABC or T_R sub-programs, and adding the temperature difference $H/A(0)$ to the results for T_R . Response to weather cycles longer

than one day must also be computed manually, as described at the end of the T_R sub-program.

R₁₂ sub-program: Run this once for each material. Angle mode must be set to 'radians'.

- 1) Enter Input: ω_0 in STO A
 $\frac{\omega_0}{\omega_w}$ in STO B * $\left(\frac{\omega_0}{\omega_w} \text{ is the period of the weather cycle in days} \right)$
 K in STO 1
 ρ_c in STO 2
 d in STO 3
 h in STO 4
 U_r in STO 5
 0 (zero) in STO I.

- 2) Press E.

Wait ~2 minutes.

Program will stop and read 'Crd' in display.

- 3) Feed in blank data card (both sides).

Note that the data card applies to this particular material.

Output:

The output R_1 and R_2 are encoded; can be interpreted with D routine of ABC program. The 'D' routine places the magnitude of R_1 or R_2 in the 'x' register and the phase angle in 'y'

*Program will not run if this storage register contains 'zero'.

$R_1(0)$ in STO 0
 $R_2(0)$ in STO 1
 $R_1(\omega_w)$ in STO 2
 $R_2(\omega_w)$ in STO 3
 $R_1(\omega_0)$ in STO 4
 $R_2(\omega_0)$ in STO 5
 $R_1(2\omega_0)$ in STO 6
 $R_2(2\omega_0)$ in STO 7
 $R_1(3\omega_0)$ in STO 8
 $R_2(3\omega_0)$ in STO 9.

ABC sub-program: Run this once for the whole building after all R_{12} data cards have been obtained. Angle mode must be set to 'radians'.

1) First: press E.

Calculator will display '8'.

2) Press 'g' MERGE (or 'f' MERGE on HP 97).

3) Read in R_{12} data card for 1st material.

4) Input h in STO A

A in STO B

α in STO C

} for the 1st material.

5) Press R/S; wait ~ 2 minutes.

Calculator will stop and display '8'.

6) Repeat steps (2) - (5) for each surface.

7) After inputting all R_{12} data cards,

enter \hat{U}_q in STO A

α_R in STO B.

Set Flag 0 (press 'h' 'SF' '0').

8) Press R/S; wait ~ 2 minutes.

Display will read 'Crd'.

9) Feed in blank data card for ABC data.

Note that output is encoded. Output for A and C can be decoded using by pressing 'd'. Output for B can be decoded by pressing 'D'. The magnitude of the response function appears in 'x' register and the phase angle in the 'y' register.

Output:

A(0)	in STO 0
B(0)	in STO 1
$A(\omega_w)$	in STO 2
$B(\omega_w)$	in STO 3
$A(\omega_0)$	in STO 4
$B(\omega_0)$	in STO 5
$A(2\omega_0)$	in STO 6
$B(2\omega_0)$	in STO 7
$A(3\omega_0)$	in STO 8
$B(3\omega_0)$	in STO 9
$C(\omega_w)$	in STO 10
$C(\omega_0)$	in STO 11.

$T_R(t)$ sub-program:

1) Read ABC data card. (This may be unnecessary if ABC has just been calculated.)

- 2) Press P ↔ S.
- 3) Enter weather data
- 0 (zero) in STO 1
- $|\Delta T_A|$ in STO 5
- t_ϕ in STO 6
- t_d^* in STO 7
- S_1 in STO 8
- $\overline{T_A}$ in STO 9
- ω_0 in STO C.

- 4) Press 'E'.

Output: After ~1 minute calculator will flash for 1 sec the time⁺ at midnight, then for 5 sec, the temperature at that time; then flash for 1 sec the time 1 hour later and flash for 5 sec the temperature at 1 a.m.; etc. Note that these temperatures do not include the effects of internal loads or supplementary heat.

On HP 97, hour will flash, T_R will be printed.

T_R also provides the following coefficients from Eq. (B11)

$$|S_1| d_0 \frac{B(0)}{A(0)} \quad \text{in} \quad \text{STO } 0$$

$$|S_1| d_0 \frac{B(\omega_w)}{A(\omega_w)} \quad \text{in} \quad \text{STO } 2 - 3^\ddagger$$

* Program may fail for $t_d = 6$ or 12 ; if there is a problem, try 6 or $12 \pm .001$.

⁺ Relative to sunrise.

[‡] Magnitude of complex number is in the first register; argument is in the second.

$$|S_1| \ d_1 \ \frac{B(\omega_0)}{A(\omega_0)} \quad \text{in} \quad \text{STO} \ 4 - 5^*$$

$$|S_1| \ d_2 \ \frac{B(2\omega_0)}{A(2\omega_0)} \quad \text{in} \quad \text{STO} \ 6 - 7^*$$

$$|S_1| \ d_3 \ \frac{B(3\omega_0)}{A(3\omega_0)} \quad \text{in} \quad \text{STO} \ 8 - 9^*$$

$$\frac{C(\omega_w)}{A(\omega_w)} \quad \text{in} \quad \text{STO} \ 10 - 11^*$$

$$\frac{C(\omega_0)}{A(\omega_0)} \quad \text{in} \quad \text{STO} \ 12 - 13^*$$

Subroutine D of T_R decodes representations of R_1 , R_2 , and B into polar format.

Subroutine d of T_R decodes representations of A and C.

Calculating response to weather cycles:

As discussed in Appendix D, only three terms need be added to the daily response calculation to compute response on any given day of a weather cycle. These terms are given in Eq. (D4) of Appendix D, "Long-term Weather Response."

To perform the computation; set $|S_1|$ equal to the amplitude of solar gain on the day in question:

$$|S_1| = \bar{S} + \Delta S_w \cos \omega_w (t-t_s)$$

where t is evaluated at noon of the day of interest.

* Magnitude of complex number is in the first register; argument is in the second.

\bar{T}_A is set equal to the average temperature for the whole cycle. The program is then run and results recorded for T_R at each hour. To these results are added the three terms from Eq. (D4).

The first term, $(\bar{S} - |S_1|) B(0)/A(0) d_0$, is computed by pressing RCL 0 and multiplying the result by $(\bar{S} - |S_1|)/|S_1|$. This temperature change is added to T_R at all hours.

The second term, $\Delta S_W B(\omega_W)/A(\omega_W) d_0 e^{i\omega_W(t-t_s)}$, is obtained as follows: Compute $\omega_W(t-t_s)$ and then enter '1'. Press RCL 3, then RCL 2. Press 'B' for complex multiplication. Then multiply the result by $\Delta S_W/|S_1|$ to get the complex number represented by this term. The temperature change is the real part of this number; Press $\rightarrow R$ and read the result from the x register. In principle, this result is different each hour of the day, but the variation is usually so slow that only two or three hours need be calculated; the rest can be obtained by linear interpolation.

The third term, $\Delta T_{A_W} C(\omega_W)/A(\omega_W) e^{i\omega_W(t-t_a)}$, is obtained similarly to the second. $\omega_W(t-t_a)$ is computed, then '1' is entered. The programmer presses $P \leftrightarrow S$, then RCL 1 and RCL 0, then (subroutine) B. The result is multiplied by ΔT_{A_W} and the real part taken by pressing $\rightarrow R$.

Addition of these three terms completes the calculation of response to long-term weather.

TI - 59 Program

This program consists of two sub-programs, PSA-1 and 2, which take input data on the building and weather and compute building response. The response is calculated at each frequency, and the results added cumulatively.

When the next term changes the results by a sufficiently small amount, usually when the frequency is greater than 3 cycles per day ($\omega > 3\omega_0$), the user terminates the calculation.

The first sub-program, PSA-1, accepts all the building parameter data and most of the weather data in its first steps. It calculates and displays the steady-state room temperature response of the building, then Fourier analyzes the solar gain function $S(t)$ into its amplitudes d_n . These results are displayed (and printed) by the calculator, and must be retained by the user for manual entry in PSA-2.

The second sub-program, PSA-2 uses the data which were stored in the memory registers by PSA-1, and the d 's generated from PSA-1. The program calculates the room temperature component at a given frequency, then adds these results to the sum of those previously computed at other frequencies. After four to five passes through this program, the results for $T_R(t)$ converge to the final answer. To get the room temperature, the user must input the steady-state term computed in PSA-1.

This program expresses the time delays somewhat differently than they are defined in the parameter section of this paper. It uses the variable ϕ to represent the phase delay (in hours) of a given weather term.

- For the ΔT_A terms, $\phi = -$ (the number of hours from midnight to the time of the temperature maximum).
- For the ΔT_{A_w} term, $\phi = -$ (the number of hours from the beginning of the temperature cycle to the time of maximum temperature).
- For the ΔS_w term $\phi = -$ (the number of hours from the beginning to the weather cycle to the maximum of solar amplitude S_1).

- For the $|S_1|$ solar terms, $\phi = -$ (the number of hours from midnight to sunrise).

Note the negative signs in the definitions of all ϕ 's.

PSA-1 sub-program instructions: (using Master Library module)

- 1) Read in program by entering 1, then pressing INV 2nd WRITE and feeding in the first side of the card. Next, enter 2, press the same keys, and read in side 2.
- 2) Choose the surface ($i=0,3$) for which you will enter the parameters. Surface 0 refers to general parameters such as $|S_1|$ and \hat{U}_q . Up to three surfaces are allowed.
 - 2a) Enter surface number (i) and press E. Calculator will display '20'.
 - 3) Enter parameter in order for surface i

(0) Enter d_i	or	$ S_1 $	Press A
(1) Enter $(\frac{1}{U_R})_i$	or	\hat{U}_q	Press A
(2) Enter h_i	or	\bar{T}_A	Press A
(3) Enter K_i	or	π/t_d	Press A
(4) Enter $(\rho c)_i$	or	H	Press A
(5) Enter α_i	or	α_R	Press A
(6) Enter A_i	or	ΔT_A	Press A

Then return to Step 2a.
 - 3b) To correct a parameter, return to step (2), and enter the number of the surface; then press E and enter the substep number shown in step 3 (e.g., '4' for ρc) and press B, then enter the correct

parameter value and press 'R/S'.

- 4) Enter the number of surfaces used; press E. Calculator will display '20'.
- 5) Press C; calculator computes $H/A(0)$ and prints result.
- 6) Press 'R/S'; calculator computes and adds steady-state ambient temperature and prints \bar{T}_A .
- 7) Enter t_d^* ; press R/S. Calculator prints $d_0 |S_1| B(0)/A(0)$.
- 8) Press 'R/S'. Calculator prints steady-state component of T_R . Record this value for future use.
- 9) Compute Fourier components of solar gain d_n for each frequency of interest 'n'.
 - a) Enter n (you will need n=1, 2, and 3 in most cases).
 - b) Press D^* .
 - c) Calculator displays $\text{Im}(d_n)$ (imaginary part of d_n). Record this value for later entry.
 - d) Press $x \leftrightarrow t$.
 - e) Calculator displays $\text{Re}(d_n)$ (real part of d_n):
Record this value for later keyboard entry.
 - f) Go back to step 9a) and enter another value of n.
- 10) Press 2nd D; calculator computes and displays d_0 .
- 11) Keep calculator on and run sub-program PSA-2.

*For $t_d = 6, 12,$ or few other values, calculator will attempt to divide by zero, resulting in an error. If this happens, re-enter t_d as .001 larger or smaller and try again.

PSA-2 sub-program instructions:

- 1) Read in PSA-2 (after having run PSA-1 to fill the calculator memories with data). Enter '1', press INV 2nd WRITE and feed in the first side of the card. Then enter 2, press the same keys, and read in side 2.
- 2) Run through steps 3 - 10 once for each frequency of interest (except $\omega=0$). Choose a frequency ω .
- 3) Computes $R_1(\omega)$ $R_2(\omega)$, and $A(\omega)$ $B(\omega)$ and $C(\omega)$.
Enter the period length in days (ω_0/ω); press A.
Calculator will display $\text{Im}(R_1(\omega))$ for the first surface.
To see $\text{Re}(R_1(\omega))$, press 'x \leftrightarrow t'.
To see $\text{Re}(R_2(\omega))$, press RCL 10.
To see $\text{Im}(R_2(\omega))$, press RCL 11.
- 4) Press R/S. (If R_1 or R_2 has been examined, press R/S twice.)
- 5) For two or more surfaces press 'R/S' to compute R_1 and R_2 for each surface. Display is as shown in Step 3.
- 6) Press R/S
- 7) Compute ambient temperature term at frequency ω . If there is no such term, go to step 9.
Enter ΔT_A or $|\Delta T_{A_w}|$.
Press B.
Calculator displays Imaginary Part of weather term. To see real part, press 'x \leftrightarrow t'. To see the term hour by hour, press 'R/S'. Calculator flashes time for 1 sec and temperature from this term for 4 sec.

- 8) Enter ϕ for the weather term, press E; calculator will display '2'.
- 9) To compute solar term at frequency ω , enter $\text{Re}(d_n)$ and press C.
 (If $\omega = \omega_w$, enter d_0). Enter $\text{Im}(d_n)$ and press 'R/S'. Enter $|S_1|$ or $|\Delta S_w|$ and press 'R/S'.
 Calculator will display imaginary part of solar term at frequency ω .
 Press 'x \leftrightarrow t' to see real part.
 To see the term hour-by-hour, press R/S; calculator flashes hour for 1 sec and solar term for 4 sec.
- 10) Enter ϕ for solar term, press E. Calculator will display '2'.
- 11) Go back to Step (2) and run through the program for another frequency.
- 12) If all frequencies of interest have been run, enter 0 and press STO 23 and STO 4. Then enter the steady-state part of T_R obtained from Step 8 of PSA-1 and press STO 3. Press E, then press D and calculator will print $T_R(t)$ for every other hour of the day beginning with midnight. If no printer is available, these 12 temperatures are found successively in RCL 48-59.

ACKNOWLEDGMENT

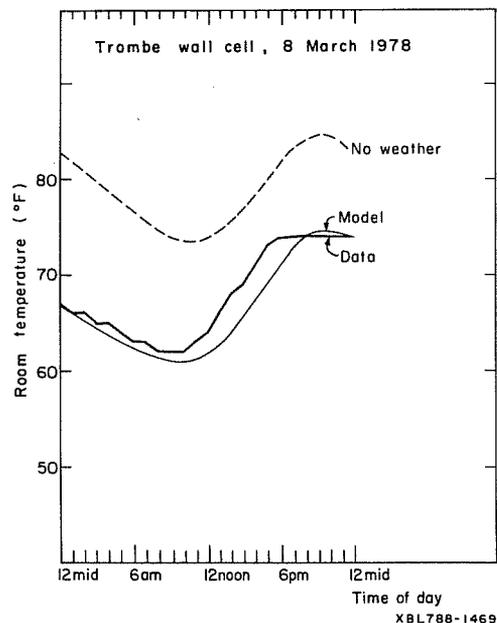
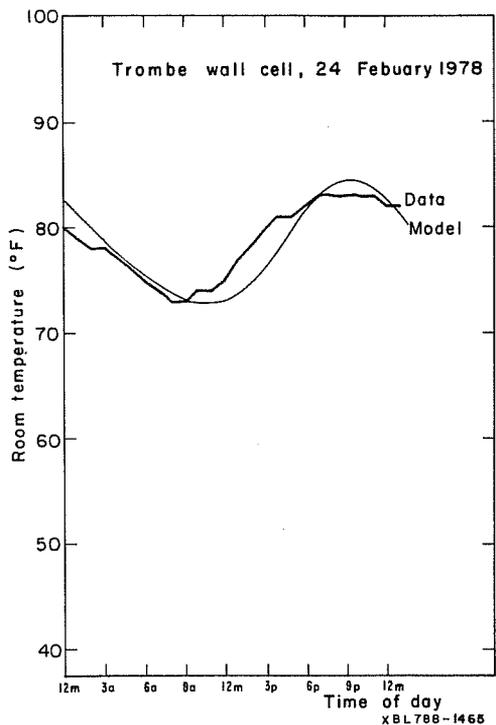
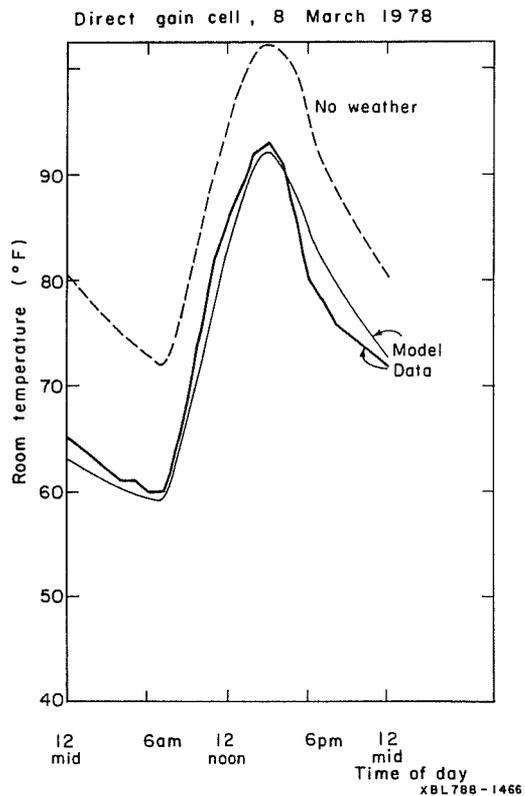
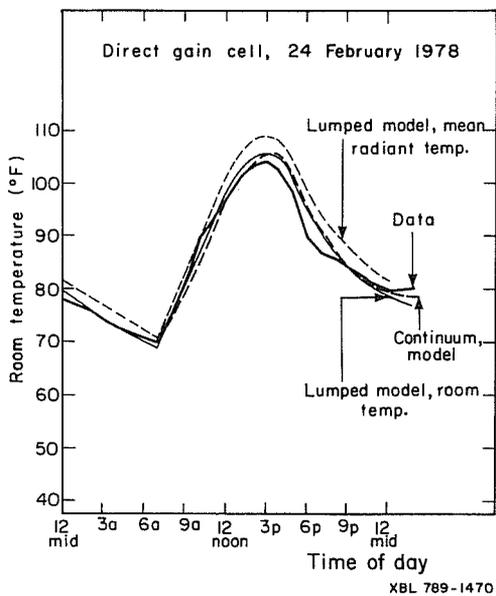
This passive solar building model was developed as part of a project on analytic building calculations at Lawrence Berkeley Laboratory, initiated by Sam Berman of LBL and Robert Richardson of New York University.

We wish to thank Ray Kinoshita for her assistance in performing the calculations in support of the frame wall model used herein. Her comments on the program operating instructions and evaluation of the parameters were also helpful in improving the usability of the program.

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2. T. Kusuda, "NBSLD, Computer Program for Heating and Cooling Loads in Buildings." Center for Building Technology, U.S. National Bureau of Standards, NBS Building Science Series 69, 1976.
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4. Metin Lokmanhekim, et al. "DOE-2: A New State-of-the-Art Computer Program for the Energy Utilization Analysis of Buildings," Presented at the Second International CIB Symposium on Energy Conservation in the Built Environment, Copenhagen, Denmark, May 28-June 1, 1979.
5. A. J. Gadgil et. al. "TWOZONE User's Manual." Lawrence Berkeley Laboratory, LBL-6840, 1978.
6. BLAST is copyrighted by the Construction Engineering Research Laboratory, U.S. Department of the Army, Champaign, Illinois.
7. J. D. Balcomb, J. C. Hedstrom, and R. D. McFarland, "Passive Solar Heating of Buildings." Los Alamos Scientific Laboratory, LA-UR-77-1162, 1977.
8. See Ref. 1, Section 3.

9. Lumen-II is a proprietary lighting analysis program written by David L. DiLaura of Smith, Hinchman, and Grylls, available for use from Computer Sharing Services, 2498 W. Second Ave., Denver, Colorado, 80223.
10. ASHRAE, Handbook of Fundamentals. 1977. Chapter 26, Tables 17-25



Predicted room temperature and observed data as a function of time of day for 24 February 1978 (left), and 8 March 1978 (right). Top: LASL direct gain cell; bottom: LASL Trombe wall cell. For 8 March, the curve labeled "no weather" assumes that all previous days had the same weather; the curve labeled "model" accounts for the previous two weeks' weather.

APPENDIX A: Listing of the R_{12} , ABC, $T_R(t)$, PSA-1, and PSA-2 Sub-Programs

R_{12}										
001	*LBLA	21	11	051	ST09	35	09	101	x	-35
002	+R		44	052	RTN		24	102	ENT↑	-21
003	ST0D	35	14	053	*LBLC	21	13	103	+P	34
004	R↓		-31	054	EEX		-23	104	ST00	35 00
005	ST0E	35	15	055	4		04	105	X≠Y	-41
006	R↓		-31	056	x		-35	106	ST0C	35 13
007	+R		44	057	INT	16	34	107	CHS	-22
008	RCLD	36	14	058	1		01	108	X≠Y	-41
009	+		-55	059	0		00	109	1/X	52
010	X≠Y		-41	060	x		-35	110	RCL4	36 04
011	RCLC	36	15	061	X≠Y		-41	111	x	-35
012	+		-55	062	*LBL2	21	02	112	RCL5	36 05
013	X≠Y		-41	063	X<0?	16-45		113	x	-35
014	+P		34	064	GT01	22	01	114	RCLC	36 13
015	RTN		24	065	+		-55	115	RCL0	36 00
016	*LBLB	21	12	066	RTN		24	116	GSBA	23 11
017	ST0D	35	14	067	*LBL1	21	01	117	RCL9	36 09
018	R↓		-31	068	2		02	118	RCL8	36 08
019	ST0E	35	15	069	Pi	16-24		119	GSBB	23 12
020	R↓		-31	070	x		-35	120	P≠S	16-51
021	RCLD	36	14	071	+		-55	121	ST08	35 08
022	x		-35	072	GT02	22	02	122	X≠Y	-41
023	X≠Y		-41	073	*LBL5	21	15	123	ST09	35 09
024	RCLC	36	15	074	RCL1	36	46	124	P≠S	16-51
025	+		-55	075	X=0?	16-43		125	0	00
026	X≠Y		-41	076	GT09	22	09	126	RCL4	36 04
027	RTN		24	077	2		02	127	RCL5	36 05
028	*LBL0	21	00	078	÷		-24	128	+	-55
029	ST0C	35	13	079	1		01	129	RCL7	36 07
030	ENT↑		-21	080	-		-45	130	RCL6	36 06
031	e ^x		33	081	3		03	131	GSBB	23 12
032	RCLC	36	13	082	X=Y?	16-33		132	P≠S	16-51
033	CHS		-22	083	SF2	16	21 02	133	RCL9	36 09
034	ENT1		-21	084	R↓		-31	134	RCL8	36 08
035	e ^x		33	085	*LBLd	21	16 14	135	P≠S	16-51
036	GSBA	23	11	086	RCLA	36	11	136	GSBA	23 11
037	2		02	087	x		-35	137	1/X	52
038	÷		-24	088	RCL2	36	02	138	X≠Y	-41
039	ST06	35	06	089	x		-35	139	CHS	-22
040	X≠Y		-41	090	RCL1	36	01	140	P≠S	16-51
041	ST07	35	07	091	÷		-24	141	ST09	35 09
042	X≠Y		-41	092	2		02	142	X≠Y	-41
043	RCLC	36	13	093	÷		-24	143	ST08	35 08
044	CHS		-22	094	IX		54	144	P≠S	16-51
045	ENT↑		-21	095	ST00	35	00	145	RCLC	36 13
046	e ^x		33	096	RCL3	36	03	146	CHS	-22
047	CHS		-22	097	x		-35	147	RCL0	36 00
048	GSBA	23	11	098	GSB0	23	00	148	1/X	52
049	ST08	35	08	099	RCL1	36	01	149	RCL5	36 05
050	X≠Y		-41	100	RCL0	36	00	150	x	-35

151	RCL9	36 09
152	RCL8	36 08
153	GSBB	23 12
154	RCL7	36 07
155	RCL6	36 06
156	GSBA	23 11
157	PzS	16-51
158	RCL9	36 09
159	RCL8	36 08
160	GSBB	23 12
161	GSBC	23 13
162	RCL9	36 09
163	RCL8	36 08
164	0	00
165	Rf	16-31
166	STOi	35 45
167	ISZI	16 26 46
168	PzS	16-51
169	CLX	-51
170	RCL5	36 05
171	GSBB	23 12
172	GSBC	23 13
173	PzS	16-51
174	STOi	35 45
175	ISZI	16 26 46
176	F2?	16 23 02
177	GT0c	22 16 13
178	PzS	16-51
179	GT0E	22 15
180	*LBL9	21 09
181	RCL1	36 01
182	RCL3	36 03
183	=	-24
184	ST08	35 08
185	1/X	52
186	RCL4	36 04
187	x	-35
188	RCL5	36 05
189	x	-35
190	RCL4	36 04
191	+	-55
192	RCL5	36 05
193	+	-55
194	ST0C	35 13
195	1/X	52
196	RCL5	36 05
197	RCL8	36 08
198	=	-24
199	1	01
200	+	-55
201	x	-35
202	RCL5	36 05
203	PzS	16-51
204	RCLC	36 13
205	=	-24
206	XzY	-41
207	0	00

208	XzY	-41
209	GSBC	23 13
210	STOi	35 45
211	ISZI	16 26 46
212	XzY	-41
213	0	00
214	XzY	-41
215	GSBC	23 13
216	STOi	35 45
217	ISZI	16 26 46
218	PzS	16-51
219	RCLB	36 12
220	1/X	52
221	GT0d	22 16 14
222	*LBLc	21 16 13
223	MDTA	16-61
224	PzS	16-51

ABC

001	*LBLA	21 11
002	+R	44
003	ST0D	35 14
004	Rf	-31
005	ST0E	35 15
006	Rf	-31
007	+R	44
008	RCLD	36 14
009	+	-55
010	XzY	-41
011	RCLC	36 15
012	+	-55
013	XzY	-41
014	+P	34
015	RTN	24
016	*LBLC	21 13
017	EEX	-23
018	4	04
019	x	-35
020	*LBL3	21 03
021	INT	16 34
022	1	01
023	0	00
024	x	-35
025	XzY	-41
026	*LBL2	21 02
027	X<0?	16-45
028	GT0I	22 01
029	+	-55
030	RTN	24
031	*LBL1	21 01
032	2	02
033	Pi	16-24
034	x	-35
035	+	-55

036	GT02	22 02
037	*LBLD	21 14
038	CF2	16 22 02
039	*LBL6	21 06
040	1	01
041	0	00
042	=	-24
043	ST0E	35 15
044	FRC	16 44
045	1	01
046	0	00
047	x	-35
048	ST0D	35 14
049	CLX	-51
050	RCLC	36 15
051	INT	16 34
052	F2?	16 23 02
053	GT04	22 04
054	EEX	-23
055	CHS	-22
056	4	04
057	x	-35
058	*LBL5	21 05
059	RCLD	36 14
060	XzY	-41
061	RTN	24
062	*LBLc	21 16 13
063	EEX	-23
064	2	02
065	x	-35
066	GT03	22 03
067	*LBLd	21 16 14
068	SF2	16 21 02
069	GT06	22 06
070	*LBL4	21 04
071	EEX	-23
072	CHS	-22
073	2	02
074	x	-35
075	GT05	22 05
076	*LBLC	21 15
077	CF0	16 22 06
078	CLRG	16-53
079	PzS	16-51
080	CLRG	16-53
081	PzS	16-51
082	*LBLc	21 16 15
083	8	08
084	ST0I	35 46
085	R/S	51
086	F0?	16 23 00
087	GT00	22 00
088	*LBL7	21 07
089	0	00
090	ST0I	35 46
091	RCLC	36 15
092	ST07	35 07

050	1/X	52	108	RTN	24	166	RCL6	36 06
051	STOA	35 11	109	*LBL0	21 00	167	-	-45
052	X=Z	-41	110	P=Z	16-51	168	RCLC	36 13
053	CHS	-22	111	RCLI	36 46	169	x	-35
054	STOB	35 12	112	3	03	170	RCL3	36 03
055	ISZI	16 26 46	113	-	-45	171	+	-55
056	GSB0	23 00	114	2	02	172	RCL2	36 02
057	RCLi	36 45	115	X>Y?	16-34	173	RCL5	36 05
058	GSBD	23 14	116	GT07	22 07	174	x	-35
059	GSBB	23 12	117	÷	-24	175	P=Z	16-51
060	P=Z	16-51	118	RCLC	36 13	176	→R	44
061	RCL8	36 08	119	x	-35	177	ST01	35 01
062	P=Z	16-51	120	STOD	35 14	178	RCLI	36 46
063	x	-35	121	CHS	-22	179	3	03
064	RCLB	36 12	122	RCL7	36 07	180	x	-35
065	RCLA	36 11	123	x	-35	181	RCLC	36 13
066	GSBB	23 12	124	1	01	182	x	-35
067	DSZI	16 25 46	125	→R	44	183	RCL9	36 09
068	SPC	16-11	126	1	01	184	+	-55
069	STOi	35 45	127	+	-55	185	RCL8	36 08
070	ISZI	16 26 46	128	→P	34	186	→R	44
071	X=Z	-41	129	RCLC	36 13	187	ST+1	35-55 01
072	STOi	35 45	130	x	-35	188	RCLI	36 46
073	9	09	131	RCL7	36 07	189	2	02
074	RCLI	36 46	132	÷	-24	190	x	-35
075	X=Y?	16-33	133	Pi	16-24	191	RCLC	36 13
076	GT09	22 09	134	RCL7	36 07	192	x	-35
077	3	03	135	÷	-24	193	RCL7	36 07
078	X=Y?	16-33	136	X²	53	194	+	-55
079	GSB0	23 00	137	RCLD	36 14	195	RCL6	36 06
080	RCLI	36 46	138	X²	53	196	→R	44
081	5	05	139	-	-45	197	ST+1	35-55 01
082	X=Y?	16-33	140	÷	-24	198	RCLI	36 46
083	GSBB	23 00	141	P=Z	16-51	199	RCLC	36 13
084	ISZI	16 26 46	142	RTN	24	200	x	-35
085	GT0a	22 16 11	143	*LBL7	21 07	201	RCL5	36 05
086	*LBL8	21 08	144	0	00	202	+	-55
087	7	07	145	RCLC	36 13	203	RCL4	36 04
088	+	-55	146	RCL7	36 07	204	→R	44
089	STOI	35 46	147	x	-35	205	ST+1	35-55 01
090	RCLi	36 45	148	Pi	16-24	206	RCL0	36 00
091	GSBd	23 16 14	149	X²	53	207	ST+1	35-55 01
092	RCLB	36 12	150	÷	-24	208	P=Z	16-51
093	RCLA	36 11	151	P=Z	16-51	209	RCL9	36 09
094	GSBB	23 12	152	RTN	24	210	P=Z	16-51
095	STOi	35 45	153	*LBL9	21 09	211	ST+1	35-55 01
096	X=Z	-41	154	P=Z	16-51	212	RCLI	36 46
097	ISZI	16 26 46	155	RCL7	36 07	213	PSE	16 51
098	RCLi	36 45	156	2	02	214	RCL1	36 01
099	X=Z	-41	157	÷	-24	215	PRTX	-14
100	STOI	35 45	158	1	01	216	ISZI	16 26 46
101	ISZI	16 26 46	159	2	02	217	SPC	16-11
102	X=Z	-41	160	-	-45	218	RCLI	36 46
103	STOi	35 45	161	STOI	35 46	219	2	02
104	RCLI	36 46	162	P=Z	16-51	220	4	04
105	9	09	163	*LBLA	21 11	221	X=Z?	16-35
106	-	-45	164	P=Z	16-51	222	RTN	24
107	STOI	35 46	165	RCLI	36 46	223	GT0A	22 11
						224	R/S	51

PSA-1

000	76	LBL	055	02	2	110	35	1/X
001	43	RCL	056	00	0	111	42	STD
002	22	INV	057	44	SUM	112	08	08
003	97	DSZ	058	01	01	113	65	x
004	09	09	059	91	R/S	114	43	RCL
005	57	ENG	060	76	LBL	115	33	33
006	03	3	061	11	A	116	95	=
007	03	3	062	72	ST*	117	44	SUM
008	42	STD	063	01	01	118	10	10
009	08	08	064	69	DP	119	01	1
010	85	+	065	21	21	120	75	-
011	53	(066	91	R/S	121	43	RCL
012	07	7	067	76	LBL	122	08	08
013	42	STD	068	12	B	123	55	+
014	07	07	069	44	SUM	124	43	RCL
015	65	x	070	01	01	125	29	29
016	43	RCL	071	91	R/S	126	95	=
017	09	09	072	72	ST*	127	65	x
018	54)	073	01	01	128	43	RCL
019	95	=	074	69	DP	129	32	32
020	42	STD	075	21	21	130	95	=
021	06	06	076	91	R/S	131	44	SUM
022	76	LBL	077	76	LBL	132	11	11
023	32	X/T	078	13	C	133	69	DP
024	73	RC*	079	43	RCL	134	29	29
025	08	08	080	00	00	135	71	SBR
026	42	STD	081	42	STD	136	43	RCL
027	05	05	082	09	09	137	69	DP
028	73	RC*	083	43	RCL	138	29	29
029	06	06	084	21	21	139	97	DSZ
030	72	ST*	085	42	STD	140	09	09
031	08	08	086	10	10	141	59	INT
032	43	RCL	087	43	RCL	142	03	3
033	05	05	088	25	25	143	07	7
034	72	ST*	089	42	STD	144	02	2
035	06	06	090	11	11	145	03	3
036	69	DP	091	76	LBL	146	69	DP
037	38	38	092	59	INT	147	04	04
038	69	DP	093	71	SBR	148	43	RCL
039	36	36	094	43	RCL	149	24	24
040	97	DSZ	095	43	RCL	150	55	+
041	07	07	096	28	28	151	43	RCL
042	32	X/T	097	85	+	152	10	10
043	76	LBL	098	43	RCL	153	95	=
044	57	ENG	099	29	29	154	42	STD
045	92	RTN	100	35	1/X	155	12	12
046	76	LBL	101	85	+	156	42	STD
047	15	E	102	53	(157	14	14
048	42	STD	103	43	RCL	158	69	DP
049	00	00	104	27	27	159	06	06
050	65	x	105	55	+	160	91	R/S
051	07	7	106	43	RCL	161	03	3
052	95	=	107	30	30	162	07	7
053	42	STD	108	54)	163	01	1
054	01	01	109	95	=	164	03	3

165	69	DP	220	03	3	275	94	+/-
166	04	04	221	07	7	276	49	PRD
167	43	RCL	222	03	3	277	02	02
168	22	22	223	05	5	278	00	0
169	44	SUM	224	00	0	279	42	STD
170	14	14	225	01	1	280	01	01
171	69	DP	226	69	DP	281	42	STD
172	06	06	227	04	04	282	04	04
173	91	R/S	228	43	RCL	283	36	PGM
174	42	STD	229	14	14	284	05	05
175	08	08	230	69	DP	285	17	B*
176	02	2	231	06	06	286	69	DP
177	03	3	232	98	ADV	287	21	21
178	69	DP	233	91	R/S	288	36	PGM
179	04	04	234	76	LBL	289	04	04
180	43	RCL	235	14	D	290	13	C
181	08	08	236	42	STD	291	03	3
182	69	DP	237	05	05	292	05	5
183	06	06	238	65	x	293	69	DP
184	35	1/X	239	89	π	294	04	04
185	65	x	240	55	+	295	43	RCL
186	89	π	241	01	1	296	01	01
187	95	=	242	02	2	297	69	DP
188	42	STD	243	95	=	298	06	06
189	23	23	244	42	STD	299	32	X!T
190	65	x	245	02	02	300	02	2
191	01	1	246	33	X²	301	04	4
192	02	2	247	94	+/-	302	69	DP
193	95	=	248	85	+	303	04	04
194	35	1/X	249	43	RCL	304	43	RCL
195	65	x	250	23	23	305	02	02
196	43	RCL	251	33	X²	306	69	DP
197	20	20	252	95	=	307	06	06
198	65	x	253	65	x	308	22	INV
199	43	RCL	254	01	1	309	37	P/R
200	11	11	255	02	2	310	99	PRT
201	55	+	256	55	+	311	32	X!T
202	43	RCL	257	43	RCL	312	99	PRT
203	10	10	258	23	23	313	98	ADV
204	95	=	259	95	=	314	32	X!T
205	42	STD	260	35	1/X	315	37	P/R
206	13	13	261	42	STD	316	91	R/S
207	44	SUM	262	03	03	317	76	LBL
208	14	14	263	01	1	318	19	D*
209	03	3	264	06	6	319	01	1
210	07	7	265	03	3	320	06	6
211	03	3	266	01	1	321	00	0
212	06	6	267	69	DP	322	01	1
213	69	DP	268	04	04	323	69	DP
214	04	04	269	43	RCL	324	04	04
215	43	RCL	270	05	05	325	43	RCL
216	13	13	271	69	DP	326	23	23
217	69	DP	272	06	06	327	65	x
218	06	06	273	43	RCL	328	01	1
219	91	R/S	274	08	08	329	02	2

330 95 =
 331 35 1/X
 332 69 DP
 333 06 06
 334 98 ADV
 335 91 R/S
 336 35 1/X
 337 99 PRT
 338 98 ADV
 339 91 R/S

PSA-2

000 76 LBL
 001 43 RCL
 002 22 INV
 003 97 DSZ
 004 09 09
 005 57 ENG
 006 03 3
 007 03 3
 008 42 STD
 009 08 08
 010 85 +
 011 53 (
 012 07 7
 013 42 STD
 014 07 07
 015 65 x
 016 43 RCL
 017 09 09
 018 54)
 019 95 =
 020 42 STD
 021 06 06
 022 76 LBL
 023 32 X:T
 024 73 RC*
 025 08 08
 026 42 STD
 027 05 05
 028 73 RC*
 029 06 06
 030 72 ST*
 031 08 08
 032 43 RCL
 033 05 05
 034 72 ST*
 035 06 06
 036 69 DP
 037 38 38
 038 69 DP
 039 36 36

040 97 DSZ
 041 07 07
 042 32 X:T
 043 76 LBL
 044 57 ENG
 045 92 RTN
 046 76 LBL
 047 65 x
 048 00 0
 049 42 STD
 050 01 01
 051 36 PGM
 052 05 05
 053 17 B*
 054 36 PGM
 055 04 04
 056 13 C
 057 92 RTN
 058 76 LBL
 059 39 CDS
 060 42 STD
 061 02 02
 062 94 +/-
 063 42 STD
 064 01 01
 065 36 PGM
 066 06 06
 067 13 C
 068 42 STD
 069 10 10
 070 42 STD
 071 12 12
 072 32 X:T
 073 42 STD
 074 11 11
 075 42 STD
 076 13 13
 077 43 RCL
 078 06 06
 079 42 STD
 080 02 02
 081 94 +/-
 082 42 STD
 083 01 01
 084 36 PGM
 085 06 06
 086 12 B
 087 94 +/-
 088 42 STD
 089 02 02
 090 32 X:T
 091 42 STD
 092 01 01
 093 92 RTN
 094 76 LBL
 095 11 A

096 35 1/X
 097 65 x
 098 89 #
 099 55 +
 100 01 1
 101 02 2
 102 95 =
 103 42 STD
 104 23 23
 105 43 RCL
 106 21 21
 107 42 STD
 108 14 14
 109 42 STD
 110 18 18
 111 43 RCL
 112 25 25
 113 42 STD
 114 16 16
 115 00 0
 116 42 STD
 117 15 15
 118 42 STD
 119 17 17
 120 42 STD
 121 19 19
 122 43 RCL
 123 00 00
 124 42 STD
 125 09 09
 126 76 LBL
 127 23 LNX
 128 71 SBR
 129 43 RCL
 130 43 RCL
 131 31 31
 132 65 x
 133 43 RCL
 134 23 23
 135 55 +
 136 43 RCL
 137 30 30
 138 55 +
 139 02 2
 140 95 =
 141 34 FX
 142 42 STD
 143 08 08
 144 65 x
 145 43 RCL
 146 28 28
 147 65 x
 148 43 RCL
 149 30 30
 150 65 x

APPENDIX B: Theory

This section describes the mathematical model which is the basis for the programs. The model looks at the response of interior (that is, room-side) surfaces of building elements, such as envelope and partition walls, on which solar heat may be absorbed. An equation is developed to describe the response of material surface temperatures to weather variables in terms of Fourier materials response functions R_1 and R_2 , which are evaluated in the programs.

Next, the room temperature response is computed from the surface temperature results. Room temperature can be given as a function of weather variables in terms of three Building Response Functions, called A, B, and C. These functions are also evaluated in the programs.

The final results for room temperature are derived from the weather description and the Building Response Functions, and hourly temperature is displayed or printed by the programs.

We now proceed to the theory.

Solar energy entering a direct-gain building passes through the room air until it reaches a surface (e.g., a floor). When it strikes the surface, some fraction is absorbed and the rest is reflected. The reflected component eventually is absorbed on some surface. The fraction of sunlight which is absorbed a given surface depends on the sun angles and room geometry in a very complex way. We assume that the internal solar radiation balance is already known, either from direct measurement or through simulation; and that the amount of sunlight absorbed on a surface "j" is given by

$\alpha_j S$, where S is the total amount of sunlight entering the building (in watts or Btu hr⁻¹).

For each surface "j", the surface temperature, T_{sj} , can be determined by a surface heat balance, which is expressed as:

$$h_j A_j (T_{sj} - T_R) - A_j K_j \left. \frac{\partial T_j(x,t)}{\partial x} \right|_{x=0} = \alpha_j S \quad (B1)$$

where

h_j is the combined radiation/convection film heat transfer coefficient for the jth surface (watts m⁻² °C⁻¹ or Btu hr⁻¹ ft⁻² °F⁻¹),

A_j is the area of the surface (m² or ft²),

T_R is the room temperature, (°C or °F),

$T_j(x,t)$ is the temperature distribution within the jth material, (°C or °F),

K_j is the conductivity of the jth material (watts m⁻¹ °C⁻¹ or Btu hr⁻¹ °F⁻¹ ft⁻¹),

α_j is the fraction of sunlight absorbed on the jth surface,

x is the distance into the material (m or ft), and

S is the solar gain transmitted through all the windows (watts or Btu/hr).

This equation sets heat losses from the surface (left-hand side of (B1)) equal to heat gains (right-hand side). It assumes that the surface transfers heat directly to the room air, rather than being in radiative contact with other surfaces, which results in a substantial simplification of the computation effort.^{B2}

Heat-flows within a material satisfy the diffusion equation:

$$K_j \frac{\partial^2 T_j(x,t)}{\partial x^2} = (\rho c)_j \frac{\partial T_j(x,t)}{\partial t} \quad (B2)$$

where $(\rho c)_j$ is the heat capacity per unit volume of the j^{th} material (watt-hr °C⁻¹m⁻³ or Btu °F⁻¹ ft⁻³).^{B3}

Equations (B1) and (B2) describe heat-flows at the inside surface of a material and in its interior; at the outside surface we assume that the material is coupled to the ambient air (at temperature T_A) by a pure resistance which can be described by a heat transfer coefficient U_r . (For an uninsulated material this coefficient is just equal to an exterior surface film coefficient).

This description allows the solution for surface temperatures in terms of the driving forces of solar gain and ambient temperature. This solution can be written in simple form if we look at the amplitudes of temperature (and solar) fluctuations at a steady harmonic frequency. The result can be expressed as:

$$T_{sj} = (h_j T_R + \alpha_j S/A_j) R_{1j} + T_A R_{2j} \quad (B3)$$

where R_{1j} and R_{2j} are frequency-dependent response functions whose forms are given in Table B1. These response functions give all the information

needed to describe the thermal behavior of the material and its surface. They are evaluated and can be displayed in the programs.

Response functions are computed for all surfaces of materials with significant thermal mass. They would be needed, for example, for surfaces of masonry materials, and also for surfaces of wood or drywall if the house is relatively lightweight. For surfaces of materials with little thermal mass, such as upholstery furniture, thin wood, carpet, insulation, etc., no computation of response functions is necessary. Solar absorption on these surfaces is accounted for by the parameter α_R , which appears below in Eq. (B4), and represents the portion of solar energy absorbed on light surfaces. If the light weight surface is an envelope wall, the U-value of the wall is computed by conventional methods. The sum of U-value times areas for all thermally light elements, including windows, is added to the heat loss rate due to infiltration, given by the heat capacity of air (0.018 Btu/°F-ft³ at sea level) times the volume of the building (in ft³) times the air change rate (in air changes per hour), to produce a single term, called \hat{U}_q , which describes all quick heat transfer. Heat losses through massive elements are already taken into account in the R_2 functions.

In most cases, only a few sets of response functions need be evaluated. For example, if both walls and floor are made of masonry with similar thermal properties, they can be combined into a single surface. No distinction need be made between directly solar-illuminated materials and those in the shade -- all that is required is that the total solar absorption on the surface be correctly specified (as a fraction of total solar heat gain S).

We combine the surface temperature results into an expression for room temperature using a heat balance for the room air. This is given by:

$$\sum_{j=1}^N \hat{h}_j (T_R - T_{sj}) + \hat{U}_q (T_R - T_A) = H + \alpha_R S \quad (B4)$$

where

$$\hat{h}_j = h_j A_j ,$$

H is the heater output,

α_R is the fraction of sunlight absorbed directly into the room air or on the surfaces of light-weight objects (e.g., upholstery),

\hat{U}_q is the quick heat transfer coefficient, the sum of U values time areas for all pure conductances (e.g., windows) plus the loss rate due to infiltration.

This heat balance says that heat losses from the room air to material surfaces plus losses directly to the outside air are equal to heat gains from the heater or from solar absorption on light material surfaces (which conduct immediately into the room air).

We can use Equations (B4) and (B3) to derive the room temperature; at any frequency, the amplitude of room temperature is given by:

$$T_R \cdot A(\omega) = S \cdot B(\omega) + T_A \cdot C(\omega) + H \quad (B5)$$

where

$$A(\omega) = \sum_{j=1}^N \hat{h}_j (1 - h_j R_{1j}) + \hat{U}_q, \quad (B6a)$$

$$B(\omega) = \sum_{j=1}^N \alpha_j h_j R_{1j} + \alpha_R, \quad \text{and} \quad (B6b)$$

$$C(\omega) = \sum_{j=1}^N \hat{h}_j R_{2j} + \hat{U}_q. \quad (B6c)$$

For the case of Trombe wall buildings, an additional term is included in each of the three building response functions; this term is not presently computed in the programs.

The building response functions are combined with weather data to give an expression for room temperature at each hour. Weather data are expressed in idealized (approximate) form, as simple sine waves. Ambient temperature is given by

$$T_A(t) = \bar{T}_A + |\Delta T_A| \cos(\omega_0(t - t_\phi)) = \bar{T}_A + \underset{\sim}{\Delta T}_A e^{i\omega_0 t} \quad (B7)$$

where

\bar{T}_A is the average temperature for the day,

$|\Delta T_A|$ is the amplitude of diurnal temperature variation,

t_ϕ is the number of hours from sunrise until the hour
at which temperature reaches its maximum,

$\underset{\sim}{\Delta T}_A$ is the complex number given by $|\Delta T_A| e^{-i\omega_0 t_\phi}$, and

$\omega_0 = 2\pi/24$ hrs.

Solar heat gain is given by a half-sine wave corresponding to a period of t_d hours of sunlight per day.

$$S(t) = \begin{cases} |S_1| \sin \omega_1 t & \text{daytime} \\ 0 & \text{night} \end{cases} \quad (\text{B8})$$

where

$$\omega_1 = \pi/t_d \quad .$$

To perform the calculations, $S(t)$ is Fourier analyzed into components at frequencies of zero, one, two, and three cycles per day. Further terms are unnecessary for two reasons: their size is smaller than the first few terms, and the building response functions reduce their effect on room temperature. This Fourier analysis allows us to express $S(t)$ as:

$$S(t) = |S_1| \sum_{n=0}^3 d_n e^{in\omega_0 t} \quad (\text{B9})$$

where $\omega_0 = 2\pi/24$ hours

and t is measured beginning at sunrise.

This expression requires that

$$d_n = \begin{cases} \frac{\omega_0 t_d}{\pi^2} & n=0 \\ \frac{\omega_0}{t_d} \frac{1 + e^{-in\omega_0 t_d}}{\left(\frac{\pi}{t_d}\right)^2 - (n\omega_0)^2} & n \neq 0 \end{cases} \quad (\text{B10})$$

The programs calculate d_n and evaluate T_R using the equation

$$T_R(t) = |S_1| \sum_{n=0}^3 \frac{B(n\omega_0)}{A(n\omega_0)} d_n e^{in\omega_0 t} + \bar{T}_A + \frac{\Delta T_A}{\sim A} \frac{C(\omega_0)}{A(\omega_0)} e^{i\omega_0 t} . \quad (B11)$$

If a heater is present, then T_R is increased by the constant $H/A(0)$ where H is the heater output and $A(0)$ turns out to equal the conventional design heat loss of the building per degree.

Notes and References

- B1. Simulating radiant energy interchange in buildings is very complex. One computer program which performs this calculation is G.P. Mitalas and D. G. Stephenson, "Fortran IV Programs to Calculate Radiant Energy Interchange Factors." Computer Program No. 25 of the Division of Building Research, National Research Council of Canada, Ottawa, Canada, 1966.
- B2. D. B. Goldstein. Some Analytic Models of Passive Solar Building Performance. Lawrence Berkeley Laboratory, LBL-7811, November, 1978, and Garland Press, New York City, 1979. See Section 2.2.5 and Appendix 2.4.
- B3. If the material consists of several layers, a separate diffusion equation is needed for each layer; but in practice, all layers beyond the first (inside) layer can usually be modeled as pure resistances, as is done in the programs.

Table B1. Equations for response functions.

Materials Response Functions R_1 and R_2 :

$$R_1(\omega) = \frac{\cosh kd + \frac{1}{Rk} \sinh kd}{\left(h + \frac{1}{R}\right) \cosh kd + \left(Kk + \frac{h}{Rk}\right) \sinh kd}$$

$$R_2(\omega) = \frac{\frac{1}{R}}{\left(h + \frac{1}{R}\right) \cosh kd + \left(Kk + \frac{h}{Rk}\right) \sinh kd}$$

where

K is the thermal conductivity of the material,

d is the thickness of the material,

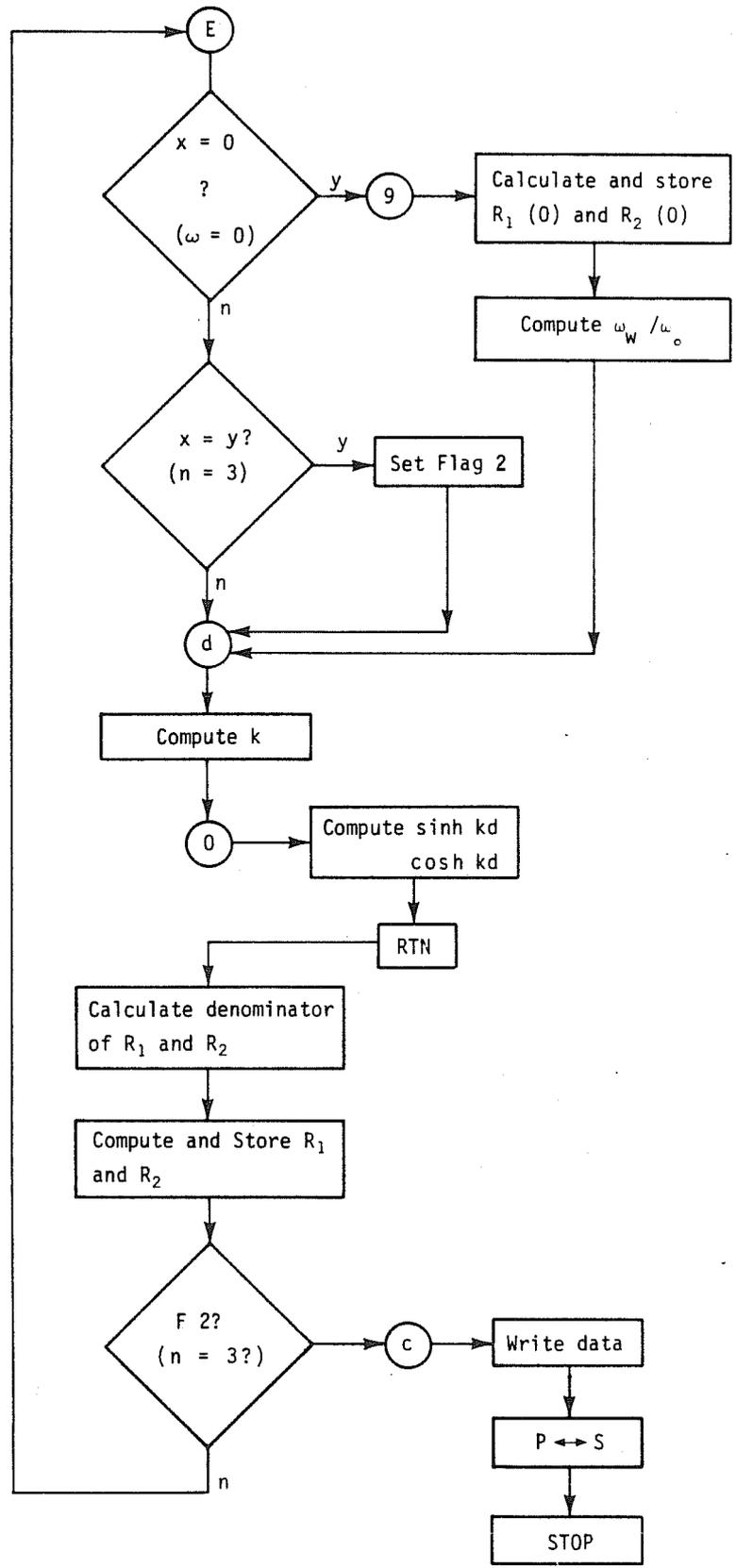
R is the thermal resistance of the exterior insulating layer, ($R = U_r^{-1}$),

$k = \sqrt{i\omega\rho c/K}$ with ρc = the volumetric heat capacity of the material, and

h is the film heat transfer coefficient for the surface.

APPENDIX C: Detailed Description of the Programs

R₁₂ Program



R₁₂ Sub-Program

Calculates $R_1(\omega)$ and $R_2(\omega)$ for a surface, at $\omega = 0, \omega_w, \omega_0, 2\omega_0, 3\omega_0$

Inputs:

STO	1	K
STO	2	ρc
STO	3	d
STO	4	h
STO	5	U_r
STO	A	ω_0
STO	B	ω_0/ω_w (or 1; can't be zero)
STO	I	0

Outputs: (encoded on data card)

STO	0	$R_1(0)$
STO	1	$R_2(0)$
STO	2	$R_1(\omega_w)$
STO	3	$R_2(\omega_w)$
STO	4	$R_1(\omega_0)$
STO	5	$R_2(\omega_0)$
STO	6	$R_1(2\omega_0)$
STO	7	$R_2(2\omega_0)$
STO	8	$R_1(3\omega_0)$
STO	9	$R_2(3\omega_0)$

Outputs are decoded with subroutine D of ABC or T_R programs. Note that outputs are calculated in registers 10-19 and switched to the primary registers for readout. Note also that $R_2(2\omega_0)$ and $R_2(3\omega_0)$ will not be used in subsequent calculations.

R₁₂ Sub-Program: Subroutines

A- Complex addition: $R_1 e^{i\theta_1} + R_2 e^{i\theta_2}$

Requires STO D and STO E

Input: x R₁

 y θ₁

 z R₂

 t θ₂

Output: complex sum (polar) R in x, θ in y.

B- Complex multiplication: $R_1 e^{i\theta_1} \times R_2 e^{i\theta_2}$

input, output, and storage requirements like A.

O- Cosh and sinh: Calculates cosh r (1+i) and sinh r (1+i) for r real. Requires STO C, D, E; calls subroutine A

Input: x r

Output: STO 6-7 cosh r (1+i) (polar)

 STO 8-9 sinh r (1+i) (polar).

C- Complex encode: Concatenates two components (R and θ) of a complex number into one number for storage. Decoding is accurate to 5-place accuracy; can be done with subroutine D of ABC and T_R programs. Valid for $R \lesssim 10$ and $\theta < 10$ radians.

Loses one significant figure for $R \gtrsim 10^{\pm 1}$; two for $R \gtrsim 10^{\pm 2}$, etc.

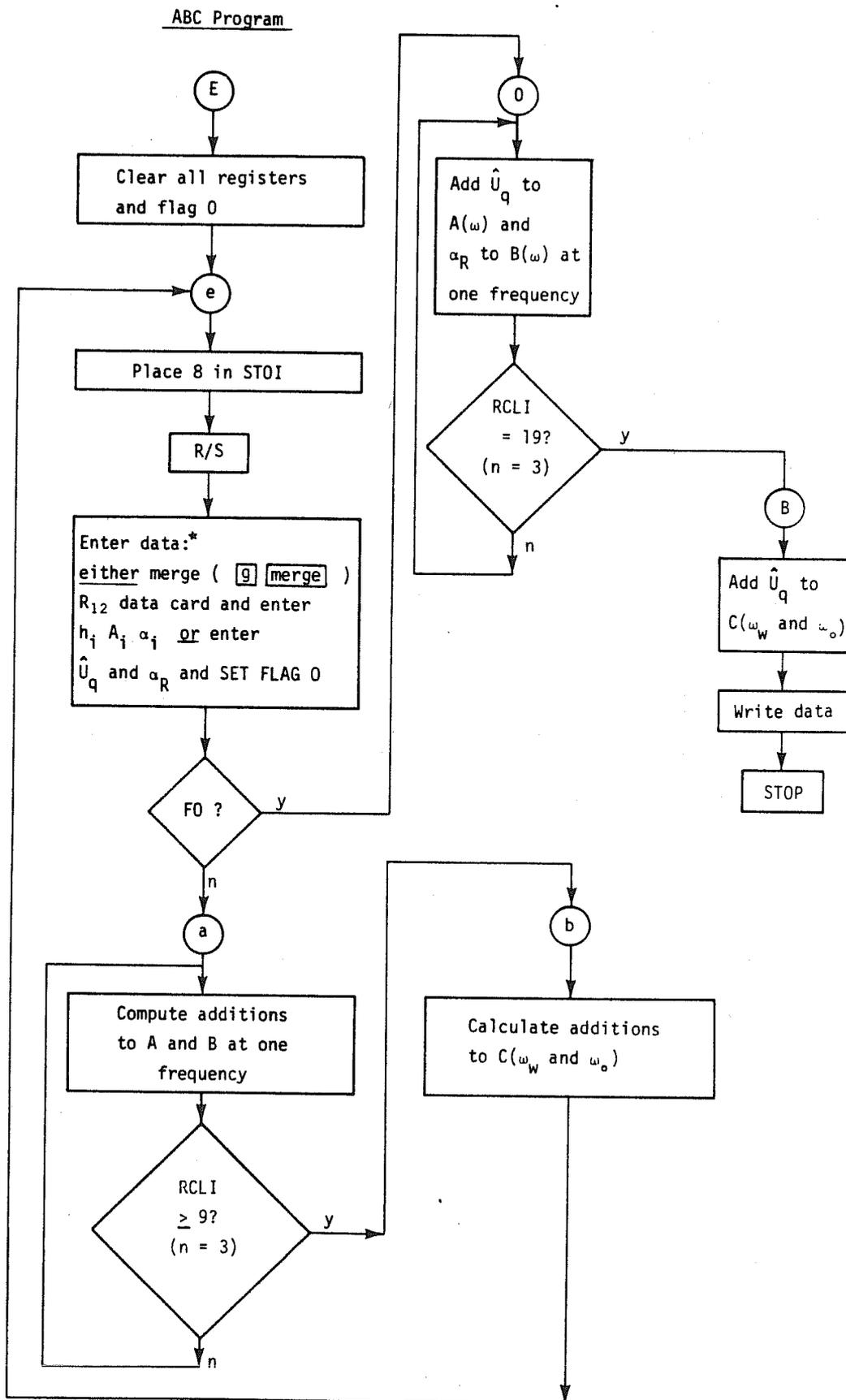
Accuracy is lost from θ for large R and from R for small R.

requires LBL 1 and LBL 2

Input: x - R

 y - θ

Output: x - encoded number.



* - performed by user

ABC Sub-Program

Calculates $A(0, \omega_w, \omega_0, 2\omega_0, 3\omega_0)$, $B(0, \omega_w, \omega_0, 2\omega_0, 3\omega_0)$, and $C(\omega_w, \omega_0)$ given R_1 and R_2 .

Inputs: R_{12} data card and h_i, A_i, α_i for each surface

Data card fills STO 0 - STO 8

STO A - h_i

STO B - A_i

STO C - α_i

Outputs: (encoded on data card)

STO 0 A(0)

STO 1 B(0)

STO 2 $A(\omega_w)$

STO 3 $B(\omega_w)$

STO 4 $A(\omega_0)$

STO 5 $B(\omega_0)$

STO 6 $A(2\omega_0)$

STO 7 $B(2\omega_0)$

STO 8 $A(3\omega_0)$

STO 9 $B(3\omega_0)$

STO 10 $C(\omega_w)$

STO 11 $C(\omega_0)$

A's and C's are decoded with 'd' subroutine; B's are decoded with 'D'.

Note that program reverses $p \leftrightarrow s$ before output is finished, so

$A(0) \rightarrow B(3\omega_0)$ are accumulated in STO 10-19.

$C(\omega_w)$ is accumulated in STO 9.

$C(\omega_0)$ is accumulated in STO 7, but transferred to STO E during data or card entry.

All storage registers are used.

ABC Sub- Program: Subroutines

A- Complex addition: $R_1 e^{i\theta_1} + R_2 e^{i\theta_2}$

requires STO D and STO E

Input: x R_1

y θ_1

z R_2

t θ_2

Output: complex sum (polar form in x-y)

C- Complex encode:

Concatenates two components (R and θ) of a complex number into one number to 5-digit accuracy.

Good for $R \lesssim 10$ and $\theta < 10$ radians.

Loses one significant figure to $R \gtrsim 10^{\pm 1}$, two for $R \gtrsim 10^{\pm 2}$, etc.

Accuracy is lost from θ for large R and from R for small R.

Requires LBL 1, LBL 2, and LBL 3.

Input: x - R

y - θ

Output: encoded number - x

C- complex encode

like C only for $R \lesssim 1000$ and $\theta < 10$.

D- Complex decode - inverse of C.

d- inverse of c.

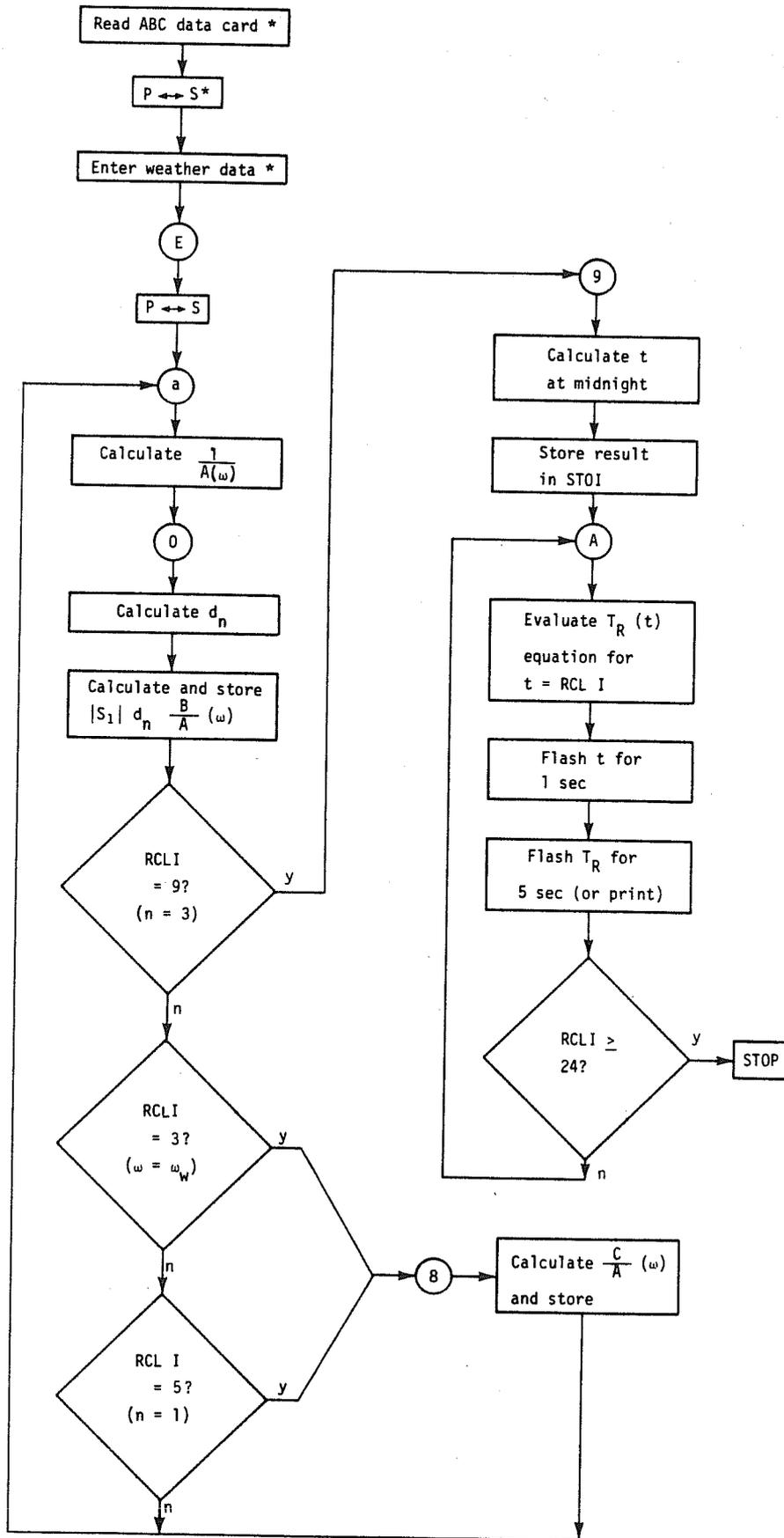
D and d both require flag 2, LBL 4, LBL 5, LBL 6, STO D, and STO E.

D and d are structured so as to preserve the contents of the y and z

stack registers. Thus, two complex numbers can be recalled from memory, decoded, and added or multiplied using these routines, as follows:

RCL 1 (recall encoded number)
D (decode and place in x and y)
RCL 2 (recall second encoded number)
D (decode and place in x-y; number from RCL 1 now in z-t)
A (add the two complex numbers)

$T_R(t)$ Program



* Performed by user

T_R(t) Sub-Program

Evaluates coefficients of $e^{in\omega_0 t}$ in the following equation:

$$T_R(t) = |S_1| \sum_{n=0}^3 \frac{B(n\omega_0)}{A(n\omega_0)} d_n e^{in\omega_0 t} + \Delta T_A \frac{C(\omega_0)}{A(\omega_0)} e^{i\omega_0 t} + \bar{T}_A$$

$$+ \left(|S_1| \frac{B(\omega_w)}{A(\omega_w)} d_0 e^{i\omega_w t} + \frac{C(\omega_w)}{A(\omega_w)} e^{i\omega_w t} \right) \quad (C1)$$

(Terms in parenthesis are not correct, but are evaluated in the form given.)

Evaluates $T_R(t)$ using Eq. (C1) (but not the part in parentheses) for each hour of the day, beginning at solar midnight.

Inputs: ABC data card fills STO 0-11

STO	I	0
STO	15	$ \Delta T_A $
STO	16	t_ϕ
STO	17	t_d
STO	18	$ S_1 $
STO	19	\bar{T}_A
STO	C	ω_0

Outputs: Time (t) at midnight flashed for 1 sec, then room temperature
 $T_R(t)$ flashed for 5 sec (or printed)
 Time (t) at 1 a.m....

Iterates for \sim 24 hours.

Outputs of $T_R(t)$:

$$\text{STO } 0 \quad |S_1| d_0 \frac{B}{A} (0)$$

$$\text{STO } 2-3 \quad |S_1| d_0 \frac{B}{A} (\omega_w)$$

$$\text{STO } 4-5 \quad |S_1| d_1 \frac{B}{A} (\omega_0)$$

$$\text{STO } 6-7 \quad |S_1| d_2 \frac{B}{A} (2\omega_0)$$

$$\text{STO } 8-9 \quad |S_1| d_3 \frac{B}{A} (3\omega_0)$$

$$\text{STO } 10-11 \quad \frac{C}{A} (\omega_w)$$

$$\text{STO } 12-13 \quad \frac{C}{A} (\omega_0)$$

Subroutines:

D- decodes complex numbers $\sim 10^0$ (e.g., $B(\omega)$)

requires flag 2, LBL 4, LBL 5, LBL 6, STO D and STO E

Input: encoded number in x

Output: complex number (polar) in x-y

d- like D; decodes complex numbers $\sim 10^2$ (e.g., $A(\omega)$, $C(\omega)$)

B- complex multiply $R_1 e^{i\theta_1} \times R_2 e^{i\theta_2}$

requires STO D, E

Input; x R_1

y θ_1

z R_2

t θ_2

Output: complex product (polar) in x-y

D- calculates d_n using contents of STO I for n,
requires STO D

Input: STO I; STO I = 1,3; calculates d_0

5 d_1

7 d_2

9 d_3

STO C - ω_0

STO 17 - t_d

Output: d_n in x-y

A- calculates $T_R(t)$ given coefficients of Eq. (B11) in STO 10 - STO 19
and STO 0 - STO 3, as described in " $T_R(t)$ Program Output" above,
with primary and secondary storage registers reversed, and

$|\Delta T_A|$ in STO 5

t_ϕ in STO 6

\bar{T}_A in STO 9

ω_0 in STO C

t in STO I

Iterates until $t > 24$ hr.

Sub-Program PSA-1

PSA-1 stores weather and building data in memories 20-46. It then computes and prints the steady-state temperature terms that arise from the heater, ambient temperature, and solar gain. The sum is printed last. Finally the Fourier components for the solar gain component can be calculated, but are not stored.

Procedure:

LBL E (46-59) sets the counter at $20 + (i \times 7)$ where i is the index of the surface (equal to 0 for the weather terms).

LBL A (60-66) stores an input parameter at the location specified by the counter and adds one to the counter.

If the user wishes to change a parameter the surface number is re-entered (LBL E) and then the parameter number and its new value and entered via LBL B (67-76). LBL B also adds one to the counter so further changes can be made through LBL A. After all parameters are entered the number of surfaces used is stored via LBL E.

LBL C (77-233) computes, stores, and prints the steady-state components from the input parameters entered in LBL E and LBL A. Steps 77-90 are used for initialization. A loop from steps 91-140 computes $A_j(0)$, $B_j(0)$ and U_j for each surface j and keeps a running sum. A subroutine call (93-94) to SBR RCL (00-45) is used to swap the locations of the j th surface parameter with those of the first surface so that direct address arithmetic can be used in the loop. A second call at 135-136 swaps the locations back again. Steps 142-233 print an identifier for each steady-state term and compute and print their values. The hours of sunshine must be entered at step 174-175 and is echoed by print-out at steps 180-183.

The formulas used in this section are:

$$A_i(0) \equiv (\text{Area})_i \left(\frac{1}{U_r} + \frac{1}{h_i} + \frac{d_i}{K_i} \right)^{-1} \quad (95-118)$$

$$A(0) = \sum_{i=1}^{i_{\max}} A_i(0) + \hat{U}_q$$

(A(0) was initialized \hat{U}_q in steps 83-86).

$$B_i(0) = (1 - U_i/h_i) \alpha_i$$

(119-132, with U_i computed in steps 110-112. U_i is the U-value of the i^{th} construction section.)

$$B(0) = \sum_{i=1}^{i_{\max}} B_i(0) + \alpha_R$$

(B(0) was initialized to α_R in steps 87-90)

$$d_0 = t_d/12\pi$$

(174-194 with input and echo to print of t_d)

$$T_H = H(0)/A(0)$$

(142-160 with print commands)

$$T_S = |S_1| d_0 B(0)/A(0) \quad (195-219 \text{ with print command})$$

The definition of the variables can be found in the input list. In addition to the above calculations LBL C also recalls and display T_A (161-173) and prints $T_R = T_H + T_A + T_S$ (220-233)

LBL D computes the Fourier components for sunshine from the following formula

$$d_n = \frac{\omega_1}{12(\omega_1^2 - (n\pi/12)^2)} (1 + e^{-in\omega_0 t_d})$$

where $\omega_0 = \pi/12$ and $\omega_1 = \pi/t_d$

n is entered in steps 236-237 and the first factor accumulated in steps 236-262. The master-library routines for complex exponentiation and multiplication are then used in steps 273-290 to compute d_n . Steps 291-316 generate the label and calculate d_n in both polar and rectangular form. Only the latter is automatically printed.

LBL D' (317-339) computes and displays the zeroth Fourier component $d_0 = \omega_0/\pi\omega_1 = t_d/12\pi$

Storage registers:

10 - ΣA_j (0)	27 d_1
11 - ΣB_j (0)	28 $(1/U_r)_1$
12 - T_H (0)	.
13 - T_S (0)	.
14 - T_R (0)	34 d_2
20 - S	.
21 - \hat{U}_q	41 d_3
	.
	.

Sub-Program PSA-2

PSA-2 calculates the frequency dependence of the response functions. From these functions the time dependent temperature terms can be calculated and summed with specified phase lags between them. The results for every other hour of a sample day are summed into memories 48-59.

Procedure:

LBL A (94-320): Computes the material response functions, and the building response functions for a specified ω . ω is calculated in steps 94-104 from the period entered by the user. After counters and initializations are performed (95-125) the imaginary and real components of the R_1 , R_2 A, B and C are calculated in the loop from 126-320. The subroutine calls 128-129 and 313-374 to SBR RCL(00-45) perform the identical purpose as in LBL C of PSA-1. The formulas for R_1 and R_2 are taken from Table B1. R_1 is calculated in steps 130-219 and 238-254.

The complex hyperbolic functions \cosh and \sinh are evaluated by a subroutine call to SBR cos (58-93) which uses the master library's complex trigonometric functions and the relationships $\cosh z = \cos iz$ and $\sinh z = -i \sin iz$. The numerator is calculated in steps 130-180 and the denominator is calculated in re-arranged form in steps 181-219. $R_1(\omega)$ is then calculated with a complex divide in steps 238-254.

$R_2(\omega)$ is calculated in steps (130-237). Since $R_2(\omega)$ is so similar to $R_1(\omega)$ the initial sums are carried out in the same location. The final complex divide is calculated in steps 220-237.

A 'R/S' is present at step 254 so that the $R(\omega)$ can be examined if it is so desired. $A(\omega)$, $B(\omega)$, and $C(\omega)$ are computed as:

$$A(\omega) = \sum_{j=1}^{j_{\max}} A_j h_j (1 - h_j R_{1j}) + \hat{U}_q \quad (273-302)$$

$$B(\omega) = \sum_{j=1}^{j_{\max}} (\alpha_j h_j R_{1j}) + \alpha_R \quad (255-272)$$

$$C(\omega) = \sum_{j=1}^{j_{\max}} (A_j h_j R_{2j}) + \hat{U}_q \quad (273-294 \text{ and } 303-310)$$

where the A_j are the j th surface areas. Steps 311-320 merely control the loop.

LBL B (410-429) computes a frequency dependent weather term;

$$T_R(\omega) = \Delta T_A C(\omega)/A(\omega).$$

The complex divide ($C(\omega)/A(\omega)$) is computed by a call (424-425) to SBR '÷' (321-339) which divides a complex number in registers 01 and 02 by $A(\omega)$ via the complex divide routine of the master library.

LBL C (430-458) computes a frequency dependent solar term:

$$\Delta T(\omega) = |S_1| \frac{d_n B(n\omega_0)}{A(n\omega_0)} \quad (438-455)$$

The real and imaginary components of d_n and $|S_1|$ are entered in steps 430-438. If a weather frequency solar term is examined, substitute the magnitude of this term for $|S_1|$ and use d_0 in place of d_n . The complex divide by $A(n\omega_0)$ is performed by a call to SBR '÷'.

Both LBL B and LBL C can call SBR PRT (375-404). SBR PRT prints hourly temperature increments over the period of the term.

LBL E (340-374): This subroutine calculates bihourly temperature increments for a given frequency dependent solar or weather term. The results are summed into registers 48-59. The phase of the term (ϕ) is entered at the beginning. The subroutine calculates the temperatures in a loop from the formula:

$$T(t) = \text{Real part} \left(\Delta T e^{i\omega(t + \phi)} \right) \quad (352-373)$$

The ΔT from LBL C or LBL D are complex numbers. LBL E calls SBR X (46-57) to perform the complex multiply.

LBL D (459-466): LBL D advances the paper and prints the contents of registers 48-59.

- A note on speeding up the program. If the program is not intended to be modified it can be noticeably sped up by replacing labels by absolute addresses. In particular, the loops to LBL SIN and LBL LIST in LBL B and LBL PRT respectively, are very slow. Delete LBL SIN (352-353) and insert one pause. Insert one step at 370 and rewrite the decrement statements as DSZ, 9, 352 (steps 370-373). Do the same with LBL LIST. Since these labels are called many times, this replacement is very noticeable in terms of program execution time.

APPENDIX D: Long Term Weather Responses

To evaluate the effect of multi-day cycles of temperature and sunlight, we assume that the cycle can be described by sinusoidal terms. Ambient temperature is taken to be of the form.

$$T_A(t) = \bar{T}_A + |\Delta T_{A_w}| e^{i\omega_w(t-t_a)} + \Delta T_{\sim A} e^{i\omega_0 t} \quad (D1)$$

where ω_w is the frequency at which weather variations take place (typically $2\pi/1$ week),

$|\Delta T_{A_w}|$ is the amplitude of weather-variation of temperature,
 t_a is the time at which the ambient temperature is at its maximum,

and \bar{T}_A is now the average temperature over the whole cycle.

Solar gain is still taken as a half sine-wave for each day, but the amplitude is assumed to be sinusoidally modulated, as shown:

$$S(t) = \begin{cases} \left(\bar{S} + \Delta S_w \cos \omega_w (t-t_s) \right) \sin \omega_1 (t-t_{sr}) & \text{day} \\ 0 & \text{night} \end{cases} \quad (D2)$$

where

\bar{S} is the average amplitude of solar gain (average $|S_1|$),

ΔS_w is the amplitude of modulation of $|S_1|$ over the cycle,

t_s is the time at which the solar energy in the cycle is at its maximum, and

T_{sr} is the time of the most recent sunrise.

It can be shown ^{D1} that room temperature is then given by:

$$\begin{aligned}
T_R(t) = & (S + \Delta S_W \cos \omega_W (t-t_s)) \sum_{n=1}^3 \frac{B(n\omega_o)}{A(n\omega_o)} d_n e^{in\omega_o t} + \bar{S} \frac{B(0)}{A(0)} d_o \\
& + \Delta S_W \frac{B(\omega_W)}{A(\omega_W)} d_o e^{i\omega_W (t-t_s)} + \bar{T}_A + \Delta T_{A_W} \frac{C(\omega_W)}{A(\omega_W)} e^{i\omega_W (t-t_a)} \\
& + \Delta T_{\sim A} \frac{C(\omega_o)}{A(\omega_o)} e^{i\omega_o t} + \frac{H}{A(0)} \quad . \quad (D3)
\end{aligned}$$

Comparing Eq. (B11) with Eq. (D1), it can be seen that if we set $S_1 = \bar{S} + \Delta S_W \cos \omega_W (t-t_s)$, Eq. (D1) says

$$\begin{aligned}
T_R(t) = & (\text{old terms}) + (\bar{S} - |S_1|) \frac{B(0)}{A(0)} d_o \\
& + \Delta S_W \frac{B(\omega_W)}{A(\omega_W)} d_o e^{i\omega_W (t-t_s)} + \Delta T_{A_W} \frac{C(\omega_W)}{A(\omega_W)} e^{i\omega_W (t-t_a)} \quad . \quad (D4)
\end{aligned}$$

Thus only three new terms and only two new combinations of Building Response functions are needed to account for multi-day weather cycles.

Notes and References

- D1. D. B. Goldstein, Some Analytic Models of Passive Solar Building Performance. Lawrence Berkeley Laboratory, LBL-7811, November, 1978, and Garland Press, New York City, 1979. See Section 2.4 and Appendix 2.4.

APPENDIX E: Example Problem Using the Program

This section sets up a simple problem of modelling a passive solar house on a sunny winter design day, and computes the input parameters to the programs. Intermediate and final results are given. The user can employ this example to check the answers he computes using the programs.

The example house is a single-story wood-frame residential structure of conventional American construction. Its only passive solar features are its bare (or tile-covered) slab-on-grade floor and its large south-facing collector window area. As the results in Table E.4 show, it should not be considered "optimized."

The use of a wood-frame house for the example illustrates some of the simplifications that can be made in using the model to describe a house. In principle, seven material surfaces would be needed for a model of this building: the stud portion of envelope walls, of partition walls, and of the ceiling; the cavity portion of envelope walls, partition walls, and ceiling; and the floor. But to increase computational speed and convenience, in this problem we use only three surfaces. This procedure can be shown to lead to less than 1⁰F errors.

Consider a 30 X 50 foot single-story house with 8-foot ceiling. The walls have R-11 insulation between 2 X 4 studs; the ceiling uses R-30 insulation and 2 X 6 joists; while the floor is a bare concrete slab on grade. There are 250 ft² of south-facing double-glazing and 30 ft² of glazing on each of the other elevations. We assume 1.5 ft² of partition wall per ft² of envelope wall.

We consider three surfaces:

- 1) Wall and ceiling studs (25% of envelope wall area,
+ 15% of partition wall area
+ 10% of ceiling area)
- 2) Wall and ceiling cavities
- 3) Floor

For the first two, we consider a wall section composed of the following layers (from inside to outside)

- 1) Inside film resistance

$R = .68 \text{ hr} \cdot \text{ft}^2 \cdot \text{°F}/\text{Btu}$ (walls)
$.61$ (ceiling)
- 2) Gypsum wall board

$K = .075 \text{ Btu}/\text{°F} \cdot \text{ft} \cdot \text{hr}$	$\rho c = 13 \text{ Btu}/\text{ft}^3 \cdot \text{°F}$
$d = 1/24 \text{ ft}$	
- 3) Insulation

$R = 11$ wall
$R = 30$ ceiling

or

- studs: $K = .068 \text{ Btu}/\text{°F} \cdot \text{ft} \cdot \text{hr}$ $\rho c = 9 \text{ Btu}/\text{ft}^3 \cdot \text{°F}$
 $d = .292$ envelope wall
 $= .146$ partition wall
 $= .458$ ceiling
- 4) (Ceiling studs only) Insulation over studs $R = 11$
 - 5) Stucco plus exterior film coefficient $R = .41$
 or attic (for the ceiling) $R \approx 3$

Areas are:

$$\begin{aligned}
 1) \text{ Studs: } & .25 \times \underbrace{(8 \times (30 + 50) \times 2)}_{\text{gross wall area}} - \underbrace{(250 + 3 \times 30)}_{\text{window area}} \\
 & + .15 \times 1.5 \times 8 \times (30 + 50) \times 2 \\
 & + .10 \times 1500 \\
 & = 235 + 288 + 150 \\
 & = 673 \text{ ft}^2
 \end{aligned}$$

- 2) Cavities $.75 \times (8 \times (30 + 50) \times 2 - (250 + 3 \times 30) + .85 \times 1.5$
 $\times 8 \times (30 + 50) \times 2 + .90 \times 1500$
 $= 705 + 1632 + 1350$
 $= 3687 \text{ ft}^2$
- 3) Floor: 1500 ft^2

For two-layer wall approximation; we average the parameters as follows:

- 1) Studs

The massive layer has the average properties of the gypsum board and the studs.

For the envelope walls, we take the weighted average of $1/K$ and ρc for $1/2''$ of gypsum and $3 \ 1/2''$ of wood

$$K_e = \left(\left(\frac{1}{2}'' \times \frac{1}{.075} + 3 \ 1/2'' + \frac{1}{.068} \right) \div 4'' \right)^{-1} = .0688$$

$$(\rho c)_e = \left(\frac{1}{2}'' \times 13 + 3 \ 1/2'' \times 9 \right) \div 4'' = 9.5$$

$$d_e = 1/3 \text{ ft}$$

For the ceiling, we take the weighted average of $1/K$ and ρc for $1/2''$ gypsum and $5 \ 1/2''$ wood:

$$K_c = \left(\frac{1}{2}'' \times \frac{1}{.075} + 5 \ 1/2'' \times \frac{1}{.068} \right) \div 6''^{-1} = .06853$$

$$(\rho c)_c = \left(\frac{1}{2}'' \times 13 + 5 \ 1/2'' \times 9 \right) \div 6'' = 9.333$$

The thickness d is $1/2$ foot

For the partition walls, we do the same average for 1 3/4" of wood (half the stud thickness)

$$K_p = (1/2" + \frac{1}{.075} \times 1\ 3/4" \times \frac{1}{.068}) \div 2\ 1/4"^{-1} = .06944$$

$$(\rho c)_p = (1/2" \times 13 + 1\ 3/4" \times 9) \div 2\ 1/4" = 9.889$$

The thickness d is 2 1/4 inches or .1875 ft

We take the area-weighted average of K, ρc , d, and h to derive the properties of the first material (the studs).

$$K_1 = (235\ \text{ft}^2 \times .0688 + 288\ \text{ft}^2 \times .06944 + 150\ \text{ft}^2 \times .06853)$$

$$\div 673\ \text{ft}^2$$

$$= .06901\ \text{Btu}/^{\circ}\text{F-hr-ft}$$

$$(\rho c)_1 = (235\ \text{ft}^2 \times 9.5 + 288\ \text{ft}^2 \times 9.889 + 150\ \text{ft}^2 \times 9.333) \div 673\ \text{ft}^2$$

$$= 9.629\ \text{Btu}/^{\circ}\text{F-ft}^3$$

$$d_1 = (235\ \text{ft}^2 \times 1/3 + 288\ \text{ft}^2 \times .1875 + 150\ \text{ft}^2 \times .5) \div 673\ \text{ft}^2$$

$$= .3081\ \text{ft}$$

$$h_1 = (235\ \text{ft}^2 \times \frac{1}{.68} + 288\ \text{ft}^2 \times \frac{1}{.68} + 150\ \text{ft}^2 \times \frac{1}{.61}) \div 673\ \text{ft}^2$$

$$= 1.508\ \text{Btu}/\text{ft}^2\ \text{-hr-}^{\circ}\text{F}$$

$$A_1 = 673\ \text{ft}^2$$

To compute U_r , we require that the steady-state heat loss through the approximate version of material 1 is the same as for the exact case.

For the exact case, the heat loss rate is

$$U_e A_e + U_c A_c$$

or

$$\begin{aligned}
 & \frac{1}{\left(\frac{.075}{1/24}\right)^{-1} + \left(\frac{.068}{.292}\right)^{-1} + .41} \times 235 \text{ ft}^2 + \frac{1}{\left(\frac{.075}{1/24}\right)^{-1} + \left(\frac{.068}{.458}\right)^{-1} + 11 + 3} \times 150 \text{ ft}^2 \\
 = & 51.22 \text{ Btu/}^{\circ}\text{F-hr}
 \end{aligned}$$

For the model, the heat loss rate is

$$\left(\left(\frac{K_1}{d_1} \right)^{-1} + U_{r1}^{-1} \right)^{-1} A_1$$

so

$$\left(\frac{A_1}{51.22} - \frac{d_1}{K_1} \right)^{-1} = U_{r1}$$

or $U_{r1} = .117 \text{ Btu/}^{\circ}\text{F-hr-ft}^2$

Note that we have ignored the inside film resistance in computing U_r for both cases.

2) Cavities

The same process is followed. The massive layer is the gypsum board and the outside layer contains everything else. Since the massive layer has only one component, the averaging process is unnecessary and K_2 , $(\rho c)_2$, and d_2 are just given by the parameters of gypsum board. A_2 is 3687 ft^2 . We calculate the average h for this surface as the area-weighted average:

$$\begin{aligned}
 h_2 = & (705 \text{ ft}^2 \times \frac{1}{.68} + 1632 \text{ ft}^2 \times \frac{1}{.68} + \\
 & 1350 \text{ ft}^2 \times \frac{1}{.61}) \div 3687 \text{ ft}^2 \\
 = & 1.532 \text{ Btu/ft}^2\text{-hr-}^{\circ}\text{F}
 \end{aligned}$$

We still have to calculate U_{r2}

For the exact case, the heat loss rate is

$$U_e A_e + U_c A_c$$

or

$$\frac{1}{\left(\frac{.075}{1/24}\right)^{-1} + 11 + .41} \times 705 \text{ ft}^2 + \frac{1}{\left(\frac{.075}{1/24}\right)^{-1} + 30 + 3} \times 1350 \text{ ft}^2$$

$$= 99.15 \text{ Btu/}^\circ\text{F-hr}$$

For the model, the heat loss rate is

$$\left(\left(\frac{K_2}{d_2} \right)^{-1} + U_r^{-1} \right)^{-1} A_2$$

so

$$\left(\frac{A_2}{99.15} - \frac{d_2}{K_2} \right)^{-1} = U_{r2}$$

or

$$U_{r2} = .0273 \text{ Btu/}^\circ\text{F-ft}^2\text{-hr}$$

3) Floor

For the floor parameters, we assume that the slab floor is coupled to the ground, and that the ground-water migration is not important in the area of the house. The mean length of a path of heat flow from the floor through the ground to the outside air is on the order of 20 feet, so, we assume a 20-foot floor thickness. (The results are not sensitive to exact floor thickness.)

We use typical materials properties for concrete (and soil) of

$$K = .8 \text{ Btu/}^{\circ}\text{F-ft-hr}, \rho c = 20 \text{ Btu/}^{\circ}\text{F-ft}^3; h = \frac{1}{.61}$$

$$= 1.639 \text{ Btu/}^{\circ}\text{F-ft}^2\text{-hr}$$

For U_r , we take the outside film coefficient at the soil, $h = 6 \text{ Btu/ft}^2\text{-}^{\circ}\text{F-hr}$, though the results would not change if h were changed from this value.

To compute the α 's, we use the "typical" values given in the text for a dark floor and light walls. Thus $\alpha_3 = .45$, $\alpha_R = .15$.

To compute α_1 , and α_2 , we average the solar gain on the envelope walls, partition walls, and ceiling over surfaces 1 and 2 on an area-weighted basis. The total solar gain absorbed on these surfaces is

$$.10 + .20 + .10 = .40, \text{ so } \alpha_1 = .40 \times \frac{673 \text{ ft}^2}{673 \text{ ft}^2 + 3687 \text{ ft}^2}$$

and

$$\alpha_2 = .40 \times \frac{3687 \text{ ft}^2}{673 \text{ ft}^2 + 3687 \text{ ft}^2} \text{ or } \alpha_1 = .062 \text{ and } \alpha_2 = .338$$

We next compute the final remaining parameter for the house, \hat{U}_q . This term is composed of infiltration losses (assumed to be at 0.6 air changes per hour) and window losses.

Infiltration loss rate is $(8 \times 30 \times 50) \text{ ft}^3 \times 0.018 \text{ Btu/}^{\circ}\text{F-ft}^3 \times 0.6 \text{ air changes/hr} = 129.6 \text{ Btu/}^{\circ}\text{F-hr}$. Window losses through double glazing with $U = 0.49 \text{ Btu/}^{\circ}\text{F-ft}^2\text{-hr}$ (1/2" air space) are given by UA or $0.49 \times (250 \text{ ft}^2 + 3 \times 30 \text{ ft}^2) = 166.6 \text{ Btu/}^{\circ}\text{F-hr}$. So $\hat{U}_q = 296.2 \text{ Btu/}^{\circ}\text{F-hr}$.

Weather Parameters

The example we take describes a northern California climate on a relatively cool but clear winter design day in January. We set $\bar{T}_A = 45^\circ$ and $\Delta T_A = 10^\circ\text{F}$, so the daily high temperature is 55 and the low is 35. We set the length of day at nine hours ($t_d = 9 \text{ hr}$). Maximum ambient temperature is at 3 pm or 7 1/2 hours after sunrise ($t_\phi = 7.5 \text{ hr}$).

Solar gain is taken from Ashrae solar heat gain tables for 40°N . latitude for January and summed over all four elevations:

S: 250 ft ² X 1630 Btu/day	=	407,500 Btu/day
E & W: 2 X 30 ft ² X 508 Btu/day	=	30,480 Btu/day
N: 30 ft ² X 118 Btu/day	=	<u>3,540 Btu/day</u>
Total		441,520 Btu/day

This total is modified by a shading coefficient of about 0.85 for double-pane glass and multiplied by 0.9 to account for opaque window frame area, so total daily solar gain entering the house is 337,760 Btu/day. S_1 is given by: (daily solar heat gain) X $\frac{\pi}{2t_d}$ so $S_1 = 58,950 \text{ Btu/hr}$.

We set $\omega_w = \frac{1}{10} \omega_0$ to calculate response functions; however, this will not be used in the calculation. To run this problem on the TI-59, the phase terms are: $\phi = -7.5$ for the solar terms and $\phi = -15$ for the temperature term.

This completes the derivation of the building and weather parameters. They are summarized in Table E.1.

Results of the Calculation

We next display intermediate results to check the operation of each of the programs. The parameters for each surface are used to compute R_1 and R_2 . Table E.2 gives the values of the response functions for each of the three surfaces, while Table E.3 gives the building response functions. The room temperature results are given in Table E.4 under the assumption of no heater output. To include the effect of 2000 Btu/hr of internal loads, add 4°F ($= 2000 \text{ Btu/hr}/A(0)$) to each entry in Table E.4. (This effect is calculated directly in the TI-59 program).

As seen in the results from Table E.4, the response of the example building is not optimal. Afternoon temperatures are too warm and morning temperatures too cool. As an alternate to this design, one could consider the effects of more insulation and smaller collector area, or more thermal mass.

Table E.1

House Parameters for Example Problem

	<u>Surface 1</u>	<u>Surface 2</u>	<u>Surface 3</u>
K	.06901	.075	0.8
ρc	9.629	13	20
d	.3081	.04167	20
h	1.508	1.532	1.639
U_r^*	.117	.0273	6
A	673	3687	1500
α	.062	.338	.45

$$\hat{U}_q = 296.2$$

$$\alpha_R = .15$$

$$\Delta T_A = 10$$

$$t_\phi = 7.5$$

$$t_d = 9$$

$$S_1 = 58950$$

$$\bar{T}_A = 45$$

* U_r^{-1} is used for input in the TI-59 program.

Table E.2
Response Functions for the Example House*

ω	R_1 (hr- ⁰ F-ft ² /Btu)		
	<u>Surface 1</u>	<u>Surface 2</u>	<u>Surface 3</u>
0	.6309	.6414	.5956
ω_0	.5473e ^{-i0.1851}	.6374e ^{-i0.0891}	.2933e ^{-i0.4385}
$2\omega_0$.5022e ^{-i0.2163}	.6258e ^{-i0.1752}	.2372e ^{-i0.5069}
$3\omega_0$.4780e ^{-i0.2443}	.608e ^{-i0.2558}	.2064e ^{-i0.54383}
		R_2	
0	.0484	.0172	.0236
ω_0	.0291e ^{-i1.2979}	.0171e ^{-i0.1281}	0
$2\omega_0$.0161e ^{-i1.9614}	.0167e ^{-i0.2531}	0
$3\omega_0$.0102e ^{-i2.4303}	.0162e ^{-i0.3725}	0

* Note that $e^x = e^{x-2\pi}$ for any x.

Table E.3
Building Response Functions for the Example House

ω	<u>A(Btu/°F-hr)</u>	<u>B</u>	<u>C(Btu/°F-hr)</u>
0	502.2	.9802	502.2 [*]
ω_0	2332.3e ^{i0.514}	.7364e ^{-i0.1783}	402.0e ^{-i0.1016}
2 ω_0	2953.0e ^{i0.562}	.6847e ^{-i0.2231}	‡
3 ω_0	3493.4e ^{i0.588}	.6504e ^{-i0.2631}	‡

*Set equal to A(0)

‡Not computed

Table E.4
Room Temperature as a Function of Time

<u>t</u>	<u>Solar Time</u>	<u>T_R (no heater or internal loads)</u>
-7.5	12 m	68.0
-6.5	1 am	67.1
-5.5	2	66.4
-4.5	3	65.9
-3.5	4	65.3
-2.5	5	64.6
-1.5	6	64.1
-.5	7	64.2
.5	8	65.5
1.5	9	68.4
2.5	10	72.7
3.5	11	77.7
4.5	12 n	82.1
5.5	1 pm	85.0
6.5	2	85.7
7.5	3	84.3
8.5	4	81.5
9.5	5	78.4
10.5	6	75.6
11.5	7	73.6
12.5	8	72.3
13.5	9	71.3
14.5	10	70.3
15.5	11	69.2
16.5	12 m	68.0