An Improved Simple Chilled Water Cooling Coil Model

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ABSTRACT
The accurate prediction of cooling and dehumidification coil performance is important in model-based fault detection and in the prediction of HVAC system energy consumption for support of both design and operations. It is frequently desirable to use a simple cooling coil model that does not require detailed specification of coil geometry and material properties. The approach adopted is to match the overall UA of the coil to the rating conditions and to estimate the air-side and water-side components of the UA using correlations developed by Holmes (1982). This approach requires some geometrical information about the coil and the paper investigates the sensitivity of the overall performance prediction to uncertainties in this information, including assuming a fixed ratio of air-side to water-side UA at the rating condition. Finally, simulation results from different coil models are compared, and experimental data are used to validate the improved cooling coil model.

INTRODUCTION
Coils in HVAC systems couple air and water (or refrigerant) loops. The accurate prediction of cooling and dehumidification coil performance is important in model-based fault detection and in the prediction of HVAC system energy consumption to support both design and operation. Simulation of transient behavior is needed to assess local loop control performance while steady state modeling is generally sufficient for energy calculations. Recent work on coil modeling has focused on modeling dynamic response (Zhou 2005, Yu et al. 2005, Yao et al. 2004, Blomberg 1999), although these studies also address aspects of steady state performance, such as the velocity dependence of the surface convective heat transfer coefficients.

A simplified method for modeling the steady state performance of cooling coils was developed by Braun (1988), who showed that the error in the heat transfer rate caused by assuming that a partially wet coil is completely wet or completely dry (whichever predicts the greater heat
transfer) does not exceed 5% over the usual range of operating conditions. One advantage of this simple model is that the parameters can be estimated from a single rating point. One disadvantage of this simple model is the discontinuities in the leaving air dry bulb temperature and moisture content at the transition between the ‘all dry’ and the ‘all wet’ approximations.

A detailed steady state model was developed by Elmahdy and Mitalas (1977); this model requires a detailed geometrical description of the coil, treats the velocity dependence of the surface convection coefficients and iterates to find the boundary between the wet and dry sections of the coil in partially wet operation. The Braun and the Elmahdy and Mitalas models are the bases of the CCSIM and CCDET models, respectively, in the ASHRAE HVAC2 Toolkit (Brandemuehl et al. 1993). Chollar and Liesen (2004) developed a model that is a hybrid of CCSIM and CCDET in that it treats partially wet operation by dividing the coil into a wet section and a dry section, eliminating the discontinuities in the output, but uses rating point conditions to define the parameters of the model.

The overall thermal resistance of a coil, \( 1/UA \), can be separated into three components: the air-side resistance, \( 1/UA_{ext} \), the resistance of the tubes, \( r_{tube} \), and the water-side resistance, \( 1/UA_{int} \) (Equation 1).

\[
\frac{1}{UA_{tot}} = \frac{1}{UA_{ext}} + r_{tube} + \frac{1}{UA_{int}}
\]

The resistance of the fins is treated as part of the air-side resistance. The tube resistance, \( r_{tube} \), is usually negligible in the case of clean coils but can be significant if the tubes are fouled.

Estimating the air-side and water-side \( UA \) values, \( UA_{ext} \) and \( UA_{int} \), is a key aspect of cooling coil modeling. The ratio of \( UA_{ext} \) to \( UA_{int} \) determines the relative humidity of the leaving air; the higher the ratio, the more saturated the air. If the model treats the dependence of the air-side and water-side convection coefficients on the fluid velocity, it is necessary to separate the overall \( UA \), \( UA_{tot} \), into its air-side and water-side components. This separation arises naturally in detailed coil models that use a geometrical description of the coil to define the model.

If a rating condition is used to define the parameters of the model and this rating point corresponds to a fully wet coil, the \( UA_{ext} \) can be determined by calculating the bypass factor from the apparatus dew-point, as described by Brandemuehl et al. (1993). If the coil is not fully wet at the rating condition, a model that treats partially wet operation by dividing the coil into a wet...
section and a dry section can be inverted numerically using an iterative method to estimate $UA_{ext}$ and $UA_{int}$ at the rating conditions. However, the uncertainty associated with the use of this method increases as the wet fraction of the coil decreases and the ratio of $UA_{ext}$ to $UA_{int}$ becomes indeterminate for a completely dry coil.

An alternative approach to estimating $UA_{ext}$ and $UA_{int}$ has been developed by Holmes (1982). Holmes made experimental measures of the performance of a number of different hot and chilled water and direct-expansion coils and presented estimated values of the coefficients $a_1$, $a_2$ and $a_3$ in Equation 2 for various types and models of cooling coil.

\[
R = a_1 S Vela^{0.8} + a_2 Velw^{0.8} + a_3
\]  

(2)

where Vela is the ratio of the volumetric flow rate of the air to the face area of the coil, $A_f$, $Vel_w$ is the speed of the water in the tubes, $S$ is ratio of the sensible and total heat transfer rates and

\[
R = \frac{A_{face} N_{row}}{UA_{int}}
\]

(3)

The values of the exponents of the speeds in Equation 2, i.e. 0.8, are the theoretical values that correspond to well-developed turbulent flow and so are more likely to be appropriate for near full load conditions than for low part load conditions.

Comparing Equation 1 and Equation 2, it might appear that corresponds to $r_{tube}$; however, Holmes found significantly non-zero values of $a_3$ for clean coils. One possible interpretation is that the effective value of the air-side exponent is less than 0.8 and a non-negligible value of $a_3$ results in a better empirical fit to the measured data. For this reason, it was decided to associate the sum of the $a_1$ and $a_3$ terms with $1/UA_{ext}$ when using Equation 2 to determine the ratio of $UA_{ext}$ to $UA_{int}$ in the model presented below.

It should be noted that some geometrical information about the coil is required to apply Equation 2, which is written in terms of the fluid speeds rather than the fluid mass flow rates that are inputs to coil models used in system simulation. The relationship between the air speed and the air mass flow rate, $m_a$, only requires the coil face area, $A_f$, and the air density, $\rho_a$:

\[
Vel_a = \frac{m_a}{\rho_a A_{face}}
\]

(4)
The relationship between the water speed and the water mass flow rate, \( m_w \), requires the tube internal diameter, \( d_i \), and the number of parallel circuits, \( N_{cir} \), in addition to the water density, \( \rho_w \):

\[
Vel_w = \frac{m_w}{\rho_w \pi \frac{d_i^2}{4} N_{cir}}
\]  

(5)

The face area, tube diameter and number of circuits can be determined more easily than the full geometrical description required by the Elmadhy and Mitalas model and CCDET, either from the manufacturer’s data or by on-site inspection. However, an even less onerous alternative, also proposed by Holmes (ibid) and applicable particularly to the stages of design before equipment selection, is to assume a fixed ratio of \( UA_{ext} \) to \( UA_{int} \) at the rating point, and this alternative is considered below.

**COOLING COIL MODEL DESCRIPTION AND ALGORITHM**

For the general case of a partially wet coil, the model treats both the dry section and the wet section as counterflow heat exchangers, with temperature as the driving potential in the dry section and enthalpy as the driving potential in the wet section. The enthalpy of the water is taken to be the enthalpy of saturated air at the temperature of the water. The UA of the dry section, \( UA_{dry} \), is given by Equation 6, (Equation 1 with \( r_{tube}=0 \)):

\[
UA_{dry} = (1 - f_{wet}) \left( \frac{1}{UA_{ext}} + \frac{1}{UA_{int}} \right)^{-1}
\]

where \( f_{wet} \) is the fraction of the coil that is wet.

UA of the wet section, \( UA_{wet} \), is given by Equation 7:

\[
UA_{wet} = f_{wet} \left( \frac{1}{UA_{ext}} \frac{c_{p,sat}}{c_p} + \frac{1}{UA_{int}} \right)^{-1}
\]

(7)

\( c_p \) is the specific heat of air and \( c_{p,sat} \) is the saturated specific heat of air at the average air temperature in the wet section. Use of \( c_{p,sat} \) enables the enthalpy-driven heat and mass transfer to be formulated in terms of temperature as the driving potential. Note that the ratio of sensible to total heat transfer, \( S \), in Equation 2 is equal to \( c_p/c_{p,sat} \) so that Equations 6 and 7 are equivalent to Equation 2, subject only to the interpretation of \( a_3 \) discussed above. It is assumed that the
sensible heat transfer coefficient of the air and the fin effectiveness each have the same value in the wet and dry sections.

One way to use the Holmes model is to determine the values of $UA_{ext}$ and $UA_{int}$ using Equations 2-5 and substitute these values into Equations 6 and 7. (Note that $S=1$ for the dry coil case (Equation 6) and, for the wet coil case, $S=c_p/c_{p, sat}$ is already included in Equation 7.) In this case, rating point information is not required and the capacity of the coil will be directly related to the capacities of the coils measured by Holmes, corrected for the face area and the number of rows.

If it is desired to match the capacity of a coil with a known rating, a better approach is to derive the overall $UA$ from the rating conditions and use Equations 2-5 and the coefficient values presented by Holmes to define the ratio of $UA_{ext}$ to $UA_{int}$. This process is now described.

The first step is to incorporate the Holmes model into Equations 6 and 7, together with a correction factor, $C$, that will be used scale the size of the coil generated by Equations 2-5 to match the duty of the target coil at the rating point:

$$UA_{ext} = CA_{face}N_{row}/(a_1 V_{el_a}^{-0.8} + a_3)$$
$$= CA_{face}N_{row}/(a_1 \left(\frac{\rho_a A_{face}}{m_a}\right)^{0.8} + a_3)$$

(8)

$$UA_{int} = CA_{face}N_{row}/(a_2 V_{el_w}^{-0.8})$$
$$= C \left(\frac{A_{face}N_{row}}{a_2}\right)\left(\frac{\pi a_1^2}{4m_w N_{cir}}\right)^{0.8}$$

(9)

There are three different modes of coil operation, which are tested for each time the model is executed. The first step is to determine if the coil is completely dry. The coil is provisionally assumed to be completely dry and Equation 6 is used with $f_{wet}=0$. If the entering dew point temperature is less than the surface temperature at the air exit, the coil is confirmed to be fully dry. If the entering dew point temperature is greater than the surface temperature at the air exit, the coil is fully or partially wet. The coil is then provisionally assumed to be fully wet and Equation 7 is used with $f_{wet}=1$. If the entering dew point temperature is greater than the surface temperature at the air entry, the coil is confirmed to be fully wet. If the entering dew point temperature is less than the surface temperature at the air entry, the coil is either partially wet and both Equation 6 and Equation 7 are used. The leaving air condition from the dry section, calculated using Equation 6, is used as the entering air condition for the wet section. The leaving
water temperature from the wet section, calculated using Equation 7, is used as the entering water temperature for the dry section. An iteration loop is used to find the value of the wet fraction, \( f_w \), for which the surface temperature in the dry section at the interface with the wet section is equal to the dew point temperature in the dry section.

Additional iteration loops are used at the beginning of the simulation to calculate the overall heat transfer coefficient at the rating point, and find the value of the correction factor, \( C \), that matches the overall heat transfer coefficient with the calculation of the UA at the rating point based on Equations 6-9. If it is important to match both the leaving dry bulb temperature and a measure of the leaving humidity and the latent duty at the rating point is comparable in magnitude to the sensible duty, separate correction factors could be introduced into Equation 8 and Equation 9. Further study would be required to determine the most robust numerical scheme.

**COOLING COIL MODEL TESTING AND SENSITIVITY ANALYSES**
The cooling coil model described here was used to model a sample cooling coil from a manufacture’s catalog. The selected coil has six rows with 20 tubes per row and 20 circuits and a face area of 1.16m\(^2\). The entering air condition is 26.7 °C dry bulb and 19.4 °C wet bulb temperature and the leaving air condition is 13.6 °C dry bulb and 13.4 °C wet bulb temperature at the rating condition. At the rating condition, the air flow rate is 2.95 m\(^3\).s\(^{-1}\), the entering water temperature is 7.2 °C, and the water flow rate is 0.00279 m\(^3\).s\(^{-1}\).

Figure 1 shows the normalized simulation results for the selected cooling coil at different air flow rates, constant water flow rate (0.001 m\(^3\).s\(^{-1}\)), all other inlet conditions being maintained at the rating condition. As expected, the leaving dry bulb temperature, which is normalized to the range between the entering water temperature and the entering air temperature, increases with increasing air flow rate. The total heat transfer rate and sensible heat transfer rate increase with increasing air flow rate while the wetted fraction of the coil decreases with increasing air flow rate.

Figure 2 shows the normalized simulation results for various water flow rates, all other inlet conditions being maintained at the rating condition. The normalized leaving dry bulb temperature decreases with increasing water flow rate. The total heat transfer rate and sensible heat transfer rate increase with increasing water flow rate while the wetted fraction of the coil increases with increasing air flow rate.
It may be difficult to determine the number of circuits by inspection in the field, particularly if the tube manifolds are covered and so it is important to understand to effect on coil performance of varying the number of circuits. Figure 3 shows the effect of varying the number of circuits while keeping the water flow rate and the size of the tubes constant.

Figure 1. Example output from a model configured with catalog data for various air flow rates
Decreasing the number of circuits by a factor of two increases the total heat transfer rate by 6% at the rating condition. This increase is mainly in the latent heat transfer, caused by a reduction in the surface temperature. Increasing the number of circuits by a factor of two decreases the total heat transfer rate by a similar amount. Decreasing the number of circuits increases the speed of the water in the tubes, thereby increasing the water-side heat transfer coefficient and hence the overall UA. This result applies to the situation in which the Holmes data are used to predict coil capacity as well as the the ratio of $UA_{ext}$ to $UA_{int}$. 

Figure 2. Example output from a model configured with catalog data for various water flow rates
As noted above, the geometrical parameters that determine the relationship between the fluid mass flow rate and the fluid velocity may be difficult to determine or may be unknown (e.g. before coil selection). In this case, it is instructive to understand the errors that would be introduced by assuming a constant ratio of $UA_{int}$ to $UA_{ext}$ at rating conditions. From Equations 8 and 9, this ratio is $a_2 Vel_w^{-0.8} : (a_1 Vel_a^{-0.8} + a_3)$. Holmes (ibid) presents values for $a_1$, $a_2$ and $a_3$ for two types of cooling coil, one with a high fin spacing and one with a low fin spacing. Typical rating point values for $Vel_a$ and $Vel_w$ are 2.5 m.s$^{-1}$ and 1.4 m.s$^{-1}$, which result in $UA_{int}$ to $UA_{ext}$ ratios of 5.15 and 3.45 for the high fin spacing and the low fin spacing, respectively. A higher fin spacing results in fewer fins, decreasing the heat transfer area on the air side and thereby decreasing the air-side UA.

COOLING COIL MODEL VALIDATION

In this section, the results of comparing the proposed cooling coil model to laboratory measurements are presented. Zhou (2005) conducted experiments on a cooling coil with eight
rows, eight tubes per row and eight circuits with 0.37 m$^2$ face area and 0.0119 m tube inside diameter. The wet coil experimental test with the highest duty was used to define the rating condition for the model. The entering air condition is 27.0°C dry bulb and 21.3°C wet bulb temperature and the leaving air condition is 15.3°C dry bulb and 14.5°C wet bulb temperature at the rating condition. The air flow rate at the rating condition is 1.07 m$^3$.s$^{-1}$, the entering water temperature is 3.1°C and the water flow rate is 0.00047 m3.s$^{-1}$. Results from the cooling coil model proposed by Chillar and Liesen (2004)—denoted by ‘CL’—are presented for comparison. The comparisons between simulation and measurements are shown in Figure 4 for the 16 dry cases and Figure 5 for the 16 wet cases (including the rating condition).

Figure 4. Comparison of the proposed model and the CL model with dry coil experiments

Figure 4 shows the comparison of the proposed model and the CL model to the dry coil experimental measurements for leaving dry bulb temperature and leaving water temperature. The comparison results indicate that the proposed model matches the experimental measurements quite closely. The maximum fractional error in the coil duty is 9.3% for the proposed model and 16.3% for CL model and the corresponding mean fractional errors (MFE) are 4.3% and 5.6% respectively. For reference, the maximum and mean fractional errors in the measured enthalpy balance between the air side and the water side are 5.1% and 3.0%.

Figure 5 shows the comparison of the proposed model and the CL model to the wet coil experimental measurements for leaving dry bulb temperature, leaving water temperature and leaving relative humidity. Again, the proposed model matches the experimental temperature measurements quite closely. The maximum fractional error in the coil duty is 7.7% for the proposed model and 24.8% for CL model and the corresponding MFE’s are 3.4% and 13.3%
respectively. The experimental relative humidity results appear to be affected by measurement problems at high relative humidity.

Analyses of the experiment data showed that the enthalpy balance between the air stream and the water stream is not exact in the wet coil cases. It was found that the imbalance is 0.1%-6.2% of the total duty, presumably due to measurement uncertainties in the experimental data (Zhou et al. 2007). On the assumption that the main source of error is the leaving relative humidity measurement, since relative humidity sensors are generally less reliable close to saturation, the measured values of the leaving relative humidity were modified so as to eliminate the enthalpy imbalance. Minor adjustments (<0.2K) were also made to the leaving air temperature where necessary to completely eliminate the imbalance. The comparison of the proposed model and the CL model to the revised leaving conditions is shown in Figure 6. The maximum fractional error in the leaving relative humidity for the proposed model is reduced to 4.7% and the MFE to 2.3%.

Figure 5. Comparison of the proposed model and the CL model with wet coil experiments
Figure 6. Comparison of the proposed model and the CL model with the revised measured leaving conditions

FIXED RATIO OF $UA_{\text{EXT}}$ TO $UA_{\text{INT}}$

As discussed above, the ratio of $UA_{\text{ext}}$ to $UA_{\text{int}}$ for typical design air and water speeds derived from Holmes’ data are 5.15 and 3.45 for the high fin spacing and the low fin spacing, respectively. Figure 6 shows a comparison of the leaving air temperatures and the leaving water temperatures for coils with the ratio of $UA_{\text{int}}$ to $UA_{\text{ext}}$ corresponding to low fin spacing coil high fin spacing. The overall $UA$ of the two coils is the same, the aim being to show the effect of assuming different values for the ratio of $UA_{\text{ext}}$ to $UA_{\text{int}}$. The Test Numbers are defined in Tab.1.

Table 1. Test Numbers used in Figure 6

<table>
<thead>
<tr>
<th>Water Flow Rate</th>
<th>100% Water Flow Rate</th>
<th>30% Water Flow Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>100% Air Flow</td>
<td>Case 1</td>
<td>Case 3</td>
</tr>
<tr>
<td>30% Air Flow</td>
<td>Case 4</td>
<td>Case 2</td>
</tr>
</tbody>
</table>
The biggest difference between the two $UA$ ratios in Figure 7 is in the leaving air temperature for Test Number 4, which has a low air flow rate and a high water flow rate. The low air capacity rate results in a large temperature change on the air side and a high sensitivity of the leaving air temperature to changes in $UA_{ext}$. For a fixed overall $UA$, the primary effect of a change in $UA_{ext}$ is to change the sensible to latent ratio, changing the leaving temperature and the leaving relative humidity while keeping the leaving enthalpy essentially constant.

The difference in leaving air temperature for Test Number 4 is 3% of the difference between the entering air temperature and the entering water temperature. This suggests that the error due to assuming an average value of the $UA$ ratio (4.3) would not be expected to exceed 1.5%, at least for the range of coils tested by Holmes and the 3.3:1 range of air flow rate found in conventional variable-air-volume (VAV) systems. It is suggested that this ratio be used in cases where there is no information about the coil geometry or configuration, e.g. when autosizing cooling coils in whole building energy simulation programs.

**CONCLUSION**

An approach to modeling the behavior of HVAC cooling coils in the absence of detailed geometrical data has been developed. The proposed model treats the wet and dry sections of a partially wet coil separately and treats the effect of varying fluid speed on the corresponding surface heat transfer coefficient. If limited geometrical data are available – the face area, the number of rows of tubes, the number of tubes per row, the tube internal diameter and
the number of water circuits – the performance of the coil, including the total duty and the sensible to latent ratio, can be estimated from empirical data published by Holmes (1982). The number of water circuits can be hard to determine in some circumstances; a factor two error in the number of circuits results in a 6% error in the total duty. The error in the sensible duty, and hence in the prediction of the leaving air temperature, is rather less but the error in the leaving humidity is correspondingly greater.

If rating point information can be used to determine the overall UA but the latent fraction is too small to provide a reliable estimate of the ratio of the air-side conductance to the water-side conductance, this ratio can be estimated from Holmes’ data using the limited geometrical data listed above. The predictions of such a model were compared to the laboratory measurements made by Zhou (2005), using one of the experimental tests to define the rating condition.

If no geometrical data are available, typical design values of the air speed and water speed can be used to establish an average value of the ratio of the water-side to the air-side conductance at full duty. A value of 4.3 for this ratio is a reasonable choice based on Holmes’ data.

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